Multi-resolution Triangulation of Scattered Data taken by 3D scanner

Bozhidar Stanchev

Abstract: A method for multi-resolution triangulation of 3D scan data is presented. Multi-resolution leads to uniform approximation where the closeness to the 3D scan data can be controlled. The presented approach has two main phases: 1. Extraction of a coarse initial mesh as first approximation of the scan data using the Marching cubes algorithm; 2. Subdivision refinement strategy that iteratively leads to an uniform multi-resolution mesh approximation of the scan data.
Key words: multi-resolution, subdivision, 3D voxel grid, marching cubes algorithm, 3D scan data

1. Introduction

3D scan systems are coming strong into practice as means of real objects geometrical modeling. They extract raw data that are not suitable for analysis. They consist of 3D points along with information about the sequence they were scanned. Scan systems can't scan a whole 3D object at once and that is why extracted data consist of a set of patches that model different parts of the object. The post-processing of scan data is going in two major directions:

- Drawing separate patches nearer to each other, i.e. reducing the noise in data. Approaches for this are called **Fine Image Registration** and the most famous of which is the ICP (Iterative Closest Point) algorithm.
- Extracting a polygon model of the scan data. **Triangle meshes** are most often used. Polygon models are convenient because of the conciseness of the data presentation and of the small (usually linear) complexity of related algorithms.

A method for extracting multi-resolution triangle mesh that approximates 3D scan data is proposed. Every triangle mesh can be only an approximation of a smooth or piecewise-smooth surface since it consists of a set of planar triangles. Approximation error that comes with the presented approach is uniformly distributed and its measure depends on a few controllable parameters.

Multi-resolution modeling techniques have certain advantages compared to other modeling approaches:

- Polygon mesh is used and this is a provident presentation from the memory usage, computation speed and algorithms' complexity points of view. It approximates a piecewise-smooth surface by a set of planar polygons and if multi-resolution is being pursued along with that then it leads to uniform error distribution of the approximation.
- A multi-resolution polygon mesh can be stored as a hierarchical structure. Upper (finer) hierarchy levels will be these polygons that subdivide lower (rougher) levels.

Subdivision techniques are based on polygon presentations and are a natural basis of the multi-resolution techniques. The presented method uses quadric-section subdivision type (a triangle is subdivided into four new ones). During the computation of new vertices computation we do not use wavelet coefficients but forces based on the mesh disposition towards the wanted surface definition (the 3D scan data).

A surface defined by a set of explicit 3D positions is called **iso-surface** (for example a point cloud or triangle mesh). Scan data can be considered as an iso-surface but just not with the needed properties to effectively process them. It is useful to encode these data in a scalar field (volumetric data grid) to have a better speed and simple related algorithms. The multi-resolution triangle mesh is extracted from such data representation. We can describe the problem as follows:

Let a scalar field as 3D volume grid is given and every knot of it has an iso-value defined (a scalar value that expresses the correlation between the knot and the surface – for example distance). Having this representation we can build a list of all grid voxels that are intersected by the surface. The purpose of iso-surface extraction is to find a triangle mesh that approximates the surface encoded in a 3D volume grid.

The method is based on forces that attract a triangle mesh towards the encoded surface. Along with that a subdivision criterion is followed. We subdivide only regions where there are not enough triangles to approximate satisfactorily the local surface according to predefined requirements. Also we apply regularization forces on the mesh vertices and they are based on the tangent mesh strains. The method leads to a triangle mesh that uniformly approximates 3D scan data and has good aspect ratio triangles.

2. Related approaches

Marching cubes method is presented by Bloomenthal [1]. Based on a surface encoded in 3D volume grid this algorithm builds surface polygonization (triangulation). Triangles' sizes depend on the grid voxel size (grid resolution). This is the main disadvantage of the method – triangles sizes do not vary accordingly to local surface features. Wood et al [2] propose a method similar to ours. Similarly they find forces that attract a mesh towards a surface encoded in 3D volume grid. There is a condition for the encoded surface – this should be closed (without boundaries). Coarse mesh as an initial approximation of the surface is extracted using surface-wave-front propagation method. It finds ribbons that describe the surface only for regions where surface topology changes. By use of all found ribbons a triangle mesh is built that is a rough description of the surface topology. Khodakovsky [3] presents an interesting progressive mesh compression approach for meshes with arbitrary topology. It is based on wavelet transformations defined on a semi-regular triangle mesh (semi-regular wavelet transforms) and reconstruction method based on subdivision (subdivision based reconstruction). Desbrun [4] as part of his work examines the computation of regularizing forces for a triangle mesh. Guskov [5] proposes a new representation approach for meshes called normal mesh. It uses triangle multi-resolution mesh where every level of subdivision consists of points that are offsets along the normal direction of the previous level. In [6] a new class is defined for piecewise-smooth surface representation based on subdivision. Loop's subdivision scheme is used. Subdivision rules are modified locally in order to model edges and creases in the model. Cohen [7] describes 3D deformable surface – deformation under the influence of outer and inner forces that attract a surface towards the boundaries of an object.

3. Method description

3.1. Encoding of the scan data into a 3D volume grid

Scan data are encoded in a 3D volume grid where for every grid knot the distance to the iso-surface is stored. Such a presentation is typical for Marching cubes polygonization algorithms.

An algorithm for encoding is proposed where as input data we have sets of 3D points (point cloud) got by a 3D scan system. These data have following characteristics:

- The 3D model consists of a set of patches, where every of them is extracted by a single movement of the scanning sensor over a part of a 3D object. The given set models a part or the whole object.
- Every patch consists a set of 3D points and the only topological connection between them is the extraction sequence.

Accuracy of 3D volume grid data representation depends on the grid thickness and this is because in one grid voxel iso-surface can be encoded only as a planar object (linear variation of the iso-surface). This is a disadvantage of 3D voxel grid representation but it provides simple processing which makes it popular:

- The 3D grid data structure allows quick index searching of a grid element and nearness between grid elements;
- Finding out if iso-surface intersects a grid voxel is also a quick operation;

Realization of the described here algorithm uses data structures proposed by Bloomenthal [1] for the Marching cubes polygonization algorithm.

3.2. Initial coarse mesh extraction (initial approximation of the iso-surface)

The Marching cubes algorithm is used to extract an initial coarse mesh as iso-surface approximation. The surface is triangulated in every grid voxel separately and these triangulations are integrated in one complete triangulation for the whole surface. Obviously the number of triangles (i.e. how rough the resultant mesh is) and how good approximation of the iso-surface is depends on the grid resolution. The main characteristics of the method are:

- Voxels adjacency in a 3D volume grid can be traced easily (by increasing/decreasing integer values in a triple (i,j,k) that presents a grid voxel).
- Iso-surface encoding does not require storing of local triangulations but signed distances, i.e. the representation consists of a set of pairs (i, F(i)) where i is a grid knot and F(i) is the signed distance to the iso-surface. Such encoding can be seen as a potential field for which we look for solution.

Easily a separation into active and passive voxels and voxel's edges can be done. Active are these that are intersected by the iso-surface.

3.3. Multi-resolution refinement strategy

Multi-resolution mesh presentation requires uniform error distribution for the error that comes along with the mesh approximation itself. For that purpose we use a criterion which says if a triangle should be subdivided. This criterion works based on the following values computed for a triangle:

- Integral of the distances from the triangle to the iso-surface along the triangle normal. Approximate computation of this integral can be found using a triangle points sampling, i.e.

$$d_{average} = \frac{\sum_{i=1}^{m_T} d_i}{m_T} \qquad (1)$$

- Triangle's curvature. Can be expressed as:

$$Curvature_T = (|K(V^1)| + |K(V^2)| + |K(V^3)|).A_T, \qquad (2)$$

where V^1, V^2 и V^3 are the three triangle's vertices, A_T is the triangle's area.

The general criterion for subdivision of a triangle is:

$$if\ ((d_{average} \geq \max\_dist)\ or\ ((Curvature \geq \max\_curvature)\ and\ (d_{average} \geq \min\_dist)))$$
$$then\ Subdivide Triangle \qquad (3)$$

Inside the criterion there are three thresholds:

max_dist – maximum distance from the triangle to the iso-surface that doesn't require subdivision;

max_curvature – maximum triangle's curvature that doesn't require subdivision;

min_dist – minimum distance from the triangle to the iso-surface under which the triangle should not be subdivided;

If the triangle's curvature requires subdivision but $d_{average} < \min\_dist$ then the triangle shouldn't be subdivided. min_dist can be assumed as a smoothing factor. The bigger the factor is, the worse approximation of the surface creases and edges (sharp features) will be, i.e. mesh approximation will be smoothed.

3.4. Outer forces attracting the mesh towards the iso-surface

We are looking for the forces that will draw the mesh towards the iso-surface. According to the Balloon method [7] the force that will be applied on a triangle will have as its direction the triangle's normal. Its sign and strength depend on the integral of distances from the triangle to the iso-surface:

$$F_T = \frac{n_T}{A_T} \cdot \int_{x \in T} d(x,c)dx\,, \qquad (4)$$

where n_T is the triangle's normal, A_T is the triangle's area and $d(x,c)$ is the distance from a triangle's point x to the iso-surface.

The described algorithm approximates the computation of this integral. First we find a set of triangle's sampling points and then consider only the distances from these points to the iso-surface. Computation accuracy depends on the thickness of the sampling.

3.5. Inner (regularizing) forces

A discrete Laplacian operator [4] for mesh smoothing (fairing) is being examined:

$$L(x_i) = \frac{1}{m} \sum_{j \in N(i)} (x_j - x_i)\,, \qquad (5)$$

where $\{x_j, j \in N(i)\}$ is the set of the neighbors of x_i in terms of the topological mesh connectivity ($N(i)$ is the set of neighbors indices in the mesh). m is the count of neighbors.

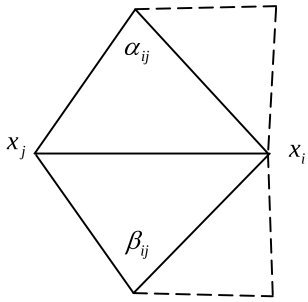
This Laplacian operator can be decomposed into two orthogonal components:

- Component that moves a mesh point x_i only along its normal vector.

The curvature normal vector can be written as:

$$K(x_i) = \frac{1}{A} \sum_{j \in N(i)} (\cot(\alpha_{ij}) + \cot(\beta_{ij})).(x_i - x_j)\,, \qquad (6)$$

where the angles α_{ij} и β_{ij} are the following:



The movement of point x_i that changes surface geometry will be with direction given by this $K(x_i)$ curvature normal vector.

- Regularizing component for the point x_i that moves it along the tangent plane.

If n is the normalized curvature normal vector then

$$T(x_i) = L(x_i) - (L(x_i).n)n \qquad (7)$$

expresses the other orthogonal component – movement along the tangent plane. This component is called **regularizing force for the point x_i in terms of the mesh**. After applying of the outer forces that approximate the mesh to the iso-surface we make mesh regularization using these regularizing forces.

3.6. Refinement strategy and subdivision criterion

- On a certain subdivision level for the mesh (i.e. for mesh with fixed triangles count and connectivity) we make minimization of a function of the distances between the mesh and the iso-surface. This is done by first applying of the outer forces which approximate the mesh to the iso-surface and after that applying mesh regularization (applying the regularizing forces).
- For every mesh triangle we determine if it should be subdivided on the next subdivision level.

The resultant mesh structure is being formed naturally as a set of tree data structures (quad-tree forest) where there is a tree structure for every triangle of the initial coarse mesh (the initial approximation of the iso-surface). Tree leafs are the triangles in the resultant mesh and branches represent the way in which the resultant mesh is build.

3.6.1. Elements of the subdivision criterion for a mesh triangle

- Iso-surface curvature in the observed triangle's area. This can be computed according to the state of the triangle in current mesh and not in terms of the 3D volume grid that encodes the iso-surface. The condition looks:

$$(|K(V^1)| + |K(V^2)| + |K(V^3)|)A_T \geq Max\_curv, \quad (8)$$

where V^1, V^2 and V^3 are the triangle's vertices. K is found according to the formula for computing of curvature normal in a mesh point (see (6)). Max_curv is the maximum curvature which does not require subdivision.

- Distance (variation) between the triangle and the iso-surface. This is just approximately computed using just a set of triangle's sampling points. Condition looks:

$$\frac{\sum_{i=1}^{m_T} d_i}{m_T} \geq Max\_dist, \qquad (9)$$

where Max_dist is the maximum distance (variation) between triangle and iso-surface which does not require subdivision.

3.6.2. Condition for preserving sharp features

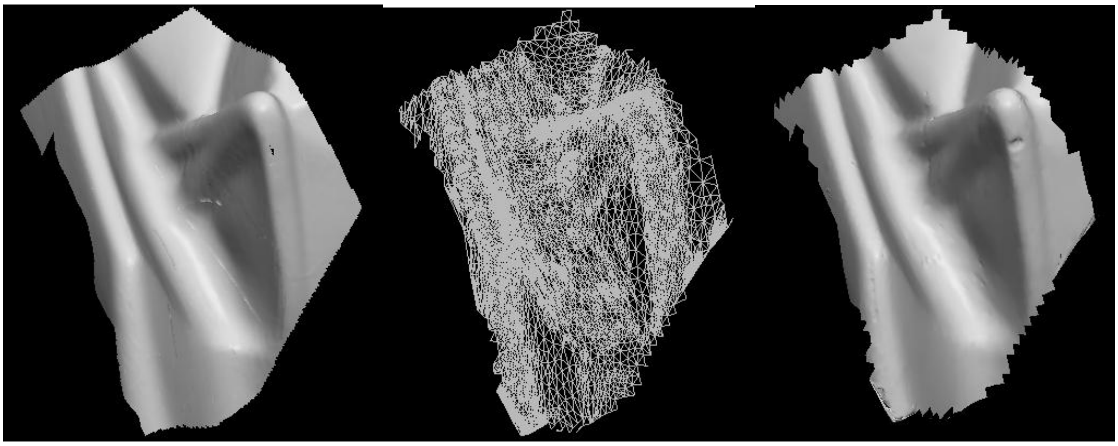
If the distance (variation) from triangle to the iso-surface is too small then the influence of the curvature in the criterion is made invalid:

$$((|K(V^1)| + |K(V^2)| + |K(V^3)|)A_T \geq Max\_curv) \; and \; \left(\frac{\sum_{i=1}^{m_T} d_i}{m_T} \geq Min\_dist\right) \qquad (10)$$

4. Results

A method for extracting multi-resolution polygon representation of 3D scan data was presented. As a result it produces multi-resolution triangle mesh with uniform approximation error distribution. The implemented program application characterizes with computational speed and optimal memory usage. The algorithm is implemented on C++ using the MS Visual Studio environment and tested on PC with processor AMD Athlon 1GHz and 256 MB RAM.

Next illustration shows what the result of the algorithm is:



A B C

Fig. 3. An example of the algorithm's work: A – scan data, B – the resultant triangle mesh with multi-resolution, C – the same triangle mesh rendered.

5. References

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About the author

Bozhidar Stanchev, PhD, phone: +359 887 44 04 86, e_mail: bstanchev@nemetschek.net