An Algorithm for Computer Synthesis of Pairs of Generalized Mutually Orthogonal Complementary Signals

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Abstract: *The present communication-information systems must satisfy large number of strong requirements, concerning their quality of service. The simultaneous providing of these requirements is possible on the base of complex radio signals with pseudo random inner structure, which autocorrelation function (ACF) has ideal shape. With regard our paper aims to suggest a mathematical algorithm for synthesis of a new class of phase manipulated (PM) signals, named pairs of generalized mutually orthogonal complementary signals. They consist of two sets of generalized complementary signals (GCSs), which are unique among all PM signals with following their features:*

- their summary (aggregated) ACF has an ideal shape, similar to a delta pulse;

- if a GCS with small length is known, then it is easy to create derivative GCSs with unlimited length;

- the aggregated cross-correlation function (CCF) of the two sets of a pair is zero everywhere.

Key words: *Synthesis, Phase Manipulated Signals, Generalized Mutually Orthogonal Complementary Signals.*

INTRODUCTION

The present communication-information systems have great impact to the improvement of the life-stile of the people in the world. This inspires great expectations for permanent enhancement of their quality of service and especially of the rate of information transmission. In order to meet these hopes the prospective communication-information systems must satisfy large number of strong requirements such as very high level of the noise immunity, effective usage of the limited natural resource – the electromagnetic spectrum and electromagnetic compatibility of the different devices. The simultaneous providing of all above requirements is possible by exploitation of complex radio signals with pseudo random inner structure, which autocorrelation function (*ACF*) has ideal shape. This means that the *ACF* of the signals is similar to the so-named delta pulse, i.e. the extreme levels of the side lobes of the *ACF* are close to zero, which guarantees maximal possible contrast between the main peak and side lobes.

Due to positive features of the above mentioned signals, the developing of the methods and algorithms for their synthesis has great theoretical and practical importance. With regard our paper aims to suggest a mathematical algorithm for synthesis of a new class of phase manipulated (*PM*) signals, named pairs of generalized mutually orthogonal complementary signals. They consist of two sets of generalized complementary signals (*GCSs*), which are unique among all *PM* signals with following their features:

- their summary (aggregated) *ACF* has an ideal shape, similar to a delta pulse;
- if a *GCS* with small length is known, then it is easy to create derivative *GCS*s with unlimited length.

Concerning the second features, it is necessary to emphasize that the most types *PM* signals with close to ideal *ACF* have limited code-length. For instance, Barker codes exist only for $n \le 13$, if n is an odd integer.

The pairs of generalized mutually orthogonal complementary signals are families of two *GCS*s, which aggregated cross-correlation function (*CCF*) of two family's members is zero everywhere.

The *GCS*s originate from the so-named Golay or complementary sequences (*CS*) [3], which are a particular case of *GCS*s, because:

- only binary phase manipulation (shift keying) is applied;
- the *CS*s are sets of only two *PM* signals.

Due to ideal shape of their *ACF*, since 70's years of the twenty century the *CS*s are widely being applied in the radar technique, radio-navigation and synchronization systems.

Moreover, in the middle of 90's it was proposed [2] the *CS*s to be used as signature signals of the users of perspective mobile communication systems, named *multi carries direct sequence code division multiple access* (*MC-DS- CDMA*) systems.

The paper is organized as follows. First, the basics of the generalized mutually orthogonal complementary signals are recalled. After that an algorithm for computer synthesis of these signals is proposed. At the end, the implementation of the presented algorithm in some areas of present communications is discussed.

AN ALGORITHM FOR COMPUTER SYNTHESIS OF PAIRS OF GENERALIZED MUTUALLY ORTHOGONAL COMPLEMENTARY SIGNALS

A. BASICS OF MUTUALLY ORTHOGONAL COMPLEMENTARY SIGNALS

It is known [5], that complex *phase manipulated (PM)* signals are sequences of *n* equivalent impulses and they are described by the formula:

$$v(t) = \sum_{j=1}^{n} U_j . u_0(t - t_j).\cos[\omega_0.(t - t_j) + \theta_j] \tag{1}$$

where τ_0 is the duration of the elementary impulses, $\omega_0 = 2\pi f_0$, f_0 is their carrier frequency, U_j - the amplitude of the j^{th} impulse and:

$$u_0(t) = \begin{cases} 1, \; if \;\; 0 \le t \le \tau_0 \\ 0, \; if \;\; t < 0, \; or \;\; t > \tau_0 \end{cases}.$$

To simplify the practical realization of the complex process of *PM* signals receiving, the following limitations in the formula (1) are made:

- $\tau_0 = const; \;\; U_j = U_0 = const; \;\; j = 1,2,...,n;$
- $\theta_j \in \{(2\pi l)/m; \;\; l = 0,1,...,m-1, (m \; is \; an \; arbitrary \; \text{int}eger)\}$.

In this case the *PM* signals can be described as a *sequence* of complex amplitudes of elementary signals [5]:

$$V(t) = \sum_{j=1}^{n} U_0 . \zeta(j) . u_0(t - t_j),$$

where $\{\zeta(j)\}_{j=0}^{n-1}$ is the set of normalized complex amplitudes of the elementary impulses:

$$\zeta(j) \in \{\exp(2\pi il/m); l = 0,1,...,m-1; \; i = \sqrt{-1}\}. \tag{2}$$

It is known [1] that a *GCS* is a *p*-set of special *PM* signals, whose summary (aggregated) non-periodical *ACF* is similar to a delta pulse (i.e. its side lobes are zero). More specifically, the following definition is used.

Definition: *The p-set of sequences:*

$$\begin{aligned} &\{A_1 = \{\xi_1(j)\}_{j=0}^{n_1-1};...;A_p = \{\xi_p(j)\}_{j=0}^{n_p-1}\}; \\ &\xi_k(j) \in \{\exp(2\pi il/m_k); l = 0,1,...,m_k - 1\}; \; k = 1,2,...,p \end{aligned}, \tag{3}$$

is called generalized complementary signal, if:

$$R_\Sigma(r) = \sum_{k=1}^{p} R_{A_k}(r) = \begin{cases} n = n_1 + n_2 + ... + n_p; if \; r = 0; \\ 0; \; if \; r = 1,2,...,\max\{n_k\}. \end{cases} \tag{4}$$

In (4) the non-periodical *ACF*s $R_{A_k}(r)$ are defined with the well known formula:

$$R_{A_k}(r) = \begin{cases} \sum_{j=0}^{n-1-|r|} \xi_k(j).\xi_k^*(j+|r|), & -(n-1) \le r \le 0; \\ \sum_{j=0}^{n-1-r} \xi_k^*(j).\xi_k(j+r), & 0 \le r \le n-1. \end{cases} \tag{5}$$

The practical implementation of *GCS*s in prospective communication-information systems depends essentially on the possibility of creating of large families of *CS*s, where every two family's members are mutually orthogonal. This means that:

$$R_{\Sigma v,w}(r) = \sum_{k=1}^{p} R_{A_{v,k},A_{w,k}}(r) = 0; \quad r = 0,1,2,...,\max\{n_k\},\ v \neq w, \tag{6}$$

where:

$$- \begin{array}{l} \{A_{s,1} = \{\xi_{s,1}(j)\}_{j=0}^{n_1-1}, ..., A_{s,p} = \{\xi_{s,p}(j)\}_{j=0}^{n_p-1}\},\ s = 1,2,...,S, \\ \xi_{s,k}(j) \in \{\exp(2\pi i l / m_k); l = 0,1,...,m_k - 1\};\ k = 1,2,...,p, \end{array} \tag{7}$$

are the sequences of a *GCS*, presenting the *s*-th member of a family of mutually orthogonal GCSs;

- S is the size of the family;
- $R_{\Sigma v,w}(r)$ is the aggregated *CCF* of the *v*-th and *w*-th family's members;

$$- R_{A_{v,k},A_{w,k}}(r) = \begin{cases} \sum_{j=0}^{n-1-|r|} \xi_{v,k}(j).\xi_{w,k}^{*}(j+|r|), & -(n-1) \le r \le 0; \\ \sum_{j=0}^{n-1-r} \xi_{v,k}^{*}(j).\xi_{w,k}(j+r), & 0 \le r \le n-1. \end{cases} \tag{8}$$

The first step toward to developing methods for creating large families of mutually orthogonal *GCS*s is finding of algorithms for synthesis of families with size $S = 2$, i.e. for synthesis of pairs of mutually orthogonal *GCS*s. This problem will be studied in more details in the rest part of the paper.

B. MAIN RESULT

Before stating the main result of this paper it should be mentioned that Tseng and Liu have proposed an algorithm (Theorem 11 in [4]) for synthesis of pairs of mutually orthogonal *CS*s. We shall generalize the result of Tseng and Liu, because they have explored only the case $m = 2$, which means usage of only binary phase manipulation (shift keying). This limitation will restrict the rate of information transmission in the perspective *MC-DS–CDMA* communication systems. Due to this reason we shall investigate the common case of *m*-ary phase manipulation with arbitrary $m \ge 2$.

Proposition: Let the *p*-set of sequences, defined by (3), be a *GCS* which size *p* is an even number and the lengths of the sequences are:

$$n_1 = n_2,\ n_3 = n_4,\ ......, n_{p-1} = n_p. \tag{9}$$

Then the *p*-set of sequences:

$$\begin{array}{l} \{B_1 = \widetilde{A}_2^{*} = \{\xi_2^{*}(j)\}_{j=n_2-1}^{0};\ B_2 = -\widetilde{A}_1^{*} = \{-\xi_1^{*}(j)\}_{j=n_2-1}^{0}, \\ B_3 = \widetilde{A}_4^{*} = \{\xi_4^{*}(j)\}_{j=n_4-1}^{0},\ B_4 = -\widetilde{A}_3^{*} = \{-\xi_3^{*}(j)\}_{j=n_4-1}^{0},\ ....., \\ B_{p-1} = \widetilde{A}_p^{*} = \{\xi_p^{*}(j)\}_{j=n_p-1}^{0};\ B_p = -\widetilde{A}_{p-1}^{*} = \{-\xi_{p-1}^{*}(j)\}_{j=n_p-1}^{0}\}, \end{array} \tag{10}$$

is a *GCS* mutually orthogonal with the *GCS*, given by (3). Here the symbols "*" and "~" mean "complex conjugating" and "reverting of the order of the sequence" respectively.

Proof: The proposition will be proved by means of so-named "method of formal polynomials (or formal power sums of a single variable)" [1]. The usage of this method is necessary, because Tseng and Liu have proved their algorithm (Theorem 11 in [4]) by combinatorial tools, based on the so-named "odd" and "even pairs". This approach is applicable only in the case $m = 2$, when the elements of the sequences (3) and (10) are -1 or +1. In the common case, when $m \ge 2$, the concept of "odd" and "even pairs" does not have sense.

According to the method of formal polynomials, let $A_k(x)$ be a polynomial, which coefficients are the elements of the sequence $A_k = \{\xi_k(j)\}_{j=0}^{n_k-1}$:

$$A_k(x) = \xi_k(0) + \xi_k(1).x + ... + \xi_k(n_k - 1).x^{n_k-1} = \sum_{j=0}^{n_k-1} \xi_k(j).x^j \,. \tag{11}$$

Then the values of the partial non-periodical *ACF* of the sequence $A_k = \{\xi_k(j)\}_{j=0}^{n_k-1}$ are the coefficients of the polynomial product $A_k(x).A_k^*(x^{-1})$, i.e.:

$$\begin{aligned} A_k(x).A_k^*(x^{-1}) = R_{A_k}(-n_k + 1).x^{-(n_k-1)} + ... + R_{A_k}(-1).x^{-1} + R_{A_k}(0) + \\ + R_{A_k}(1).x + ... + R_{A_k}(n_k - 1).x^{n_k-1}. \end{aligned} \tag{12}$$

Here $A_k^*(x^{-1})$ is the polynomial:

$$A_k^*(x^{-1}) = \xi_k^*(0) + \xi_k^*(1).x^{-1} + ... + \xi_k^*(n_k - 1).x^{-(n_k-1)} = \sum_{j=0}^{n_k-1} \xi_k^*(j).x^{-j} \,. \tag{13}$$

With regard to (12), the definition (4) of the *GCS*s can be presented in the form:

$$\sum_{k=1}^{p} A_k(x).A_k^*(x^{-1}) = n_1 + n_2 + ... + n_p \,. \tag{14}$$

In order to prove the proposition first of all it should be shown that the *p*-set of sequences (10) is a *GCS*. This will be done by evaluating of the values of the aggregated *ACF* of the *p*-set (10). According to (12), these values are the coefficients of the polynomial:

$$\sum_{k=1}^{p} B_k(x).B_k^*(x^{-1}) = \sum_{k=1}^{p} \left[(-1)^k \widetilde{A}_k^*(x)\right].\left[(-1)^k \widetilde{A}_k^*(x^{-1})\right]^* = \sum_{k=1}^{p} \widetilde{A}_k^*(x).\widetilde{A}_k(x^{-1}) \,. \tag{15}$$

Now it should be seen that:

$$\begin{aligned} \widetilde{A}_k^*(x) = \xi_k^*(0).x^{n_k-1} + \xi_k^*(1).x^{n_k-2} + ... + \xi_k^*(n_k - 2).x + \xi_k^*(n_k - 1) = \\ = x^{n_k-1}\left[\sum_{j=0}^{n_k-1} \xi_k^*(j).x^{-j}\right] = x^{n_k-1}.A_k^*(x^{-1}). \end{aligned} \tag{16}$$

The accounting of (16) in (15) leads to:

$$\sum_{k=1}^{p} B_k(x).B_k^*(x^{-1}) = \sum_{k=1}^{p} \left[x^{n_k-1}.A_k^*(x^{-1})\right]\widetilde{A}_k(x^{-1}) \,. \tag{17}$$

But:

$$\begin{aligned} x^{n_k-1}\widetilde{A}_k(x^{-1}) = x^{n_k-1}.[\xi_k(0).x^{-(n_k-1)} + ... + \xi_k(n_k - 2).x^{-1} + \xi_k(n_k - 1)] = \\ = \sum_{j=0}^{n_k-1} \xi_k(j).x^j = A_k(x). \end{aligned} \tag{18}$$

Having in mind (4) and (9), the Eq. (15) can be presented in the form:

$$\sum_{k=1}^{p} B_k(x).B_k^*(x^{-1}) = \sum_{k=1}^{p} A_k(x)A_k^*(x^{-1}) = n_1 + n_2 + ... + n_p = 2(n_2 + n_4 + ... + n_p) \,. \tag{19}$$

Consequently, the *p*-set (10) is a *GCS* according to (14).

Now we shall show that the aggregated *CCF* of the *GCS*s (3) and (10) is zero everywhere.

According to the method of formal polynomials, the values of the partial *CCF* of the *k*-th sequences of the *GCS*s (3) and (10) are the coefficients of the polynomial product $A_k(x).B_k^*(x^{-1})$. Then the values of aggregated *CCF* of *GCS*s (3) and (10) are the coefficients of the polynomial sum:

$$\sum_{k=1}^{p} A_k(x).B_k^*(x^{-1}) \,. \tag{20}$$

With regard to (10) the sum (20) can be evaluated as follows:

$$\sum_{k=1}^{p} A_k(x).B_k^*(x^{-1}) = \sum_{k=1}^{\frac{p}{2}} \left\{ A_{2k-1}(x).\left[\widetilde{A}_{2k}^*(x^{-1})\right]^* + A_{2k}(x).\left[-\widetilde{A}_{2k-1}^*(x^{-1})\right]^* \right\} = \sum_{k=1}^{\frac{p}{2}} \left[A_{2k-1}(x).\widetilde{A}_{2k}(x^{-1}) - A_{2k}(x).\widetilde{A}_{2k-1}(x^{-1}) \right] \quad (21)$$

Analogously to (15), (16) and (17) one can easily show that:

$$A_{2k}(x).\widetilde{A}_{2k-1}(x^{-1}) = \widetilde{A}_{2k}(x^{-1}).A_{2k-1}(x). \quad (22)$$

The accounting of Eq. (22) in (21) leads to:

$$\sum_{k=1}^{p} A_k(x).B_k^*(x^{-1}) = 0, \quad (23)$$

which should be proved.

The algorithm for synthesis of pairs of generalized mutually orthogonal complementary signals will be explained by following example.

Example: It is easy to verify that *2*-set of sequences:

$$\left\{ A_1 = \{1, 1, -1\};\ A_2 = \{1, i, 1\},\quad i = \sqrt{-1} \right\}, \quad (24)$$

is a *GCS*. The partial *ACF*s of the sequences A_1, A_2 and their aggregated *ACF* are presented in the upper three rows of the Table I.

After applying of the construction of the above proved proposition one can easily obtain the set of sequences:

$$\left\{ B_1 = \widetilde{A}_2^* = \{1, -i, 1\};\ B_2 = -\widetilde{A}_1^* = \{1, -1, -1\} \right\}. \quad (25)$$

The partial *ACF*s of the sequences B_1, B_2 and their aggregated *ACF* are shown in last three rows of the Table I. They demonstrate that *2*-set of sequences $\{B_1, B_2\}$ is also a *GCS*.

Table I

ACFs of the GCSs of the pair $\{(A_1, A_2), (B_1, B_2)\}$ of generalized mutually orthogonal complementary signals

ACF	**Time shift (r)**				
	-2	**-1**	**0**	**1**	**2**
R_{A_1}	**-1**	**0**	**3**	**0**	**-1**
R_{A_2}	**1**	**0**	**3**	**0**	**1**
R_{Σ_A}	**0**	**0**	**6**	**0**	**0**
R_{B_1}	**1**	**0**	**3**	**0**	**1**
R_{B_2}	**-1**	**0**	**3**	**0**	**-1**
R_{Σ_B}	**0**	**0**	**6**	**0**	**0**

The partial *CCF*s of the sequences (A_1, B_1), (A_2, B_2) and their aggregated *CCF* are presented in the Table II. The last row of the Table II shows that the family, consisting of the *GCS*s $\{A_1, A_2\}$ and $\{B_1, B_2\}$, is a pair of generalized mutually orthogonal complementary signals.

Table II

CCFs of the GCSs of the pair $\{(A_1, A_2), (B_1, B_2)\}$ *of generalized mutually orthogonal complementary signals*

CCF	**Time shift (r)**				
	-2	**-1**	**0**	**1**	**2**
$R_{A_1B_2}$	**1**	**1+i**	**i**	**1-i**	**-1**
$R_{A_2B_1}$	**-1**	**-1-i**	**-i**	**-1+i**	**1**
$R_{\Sigma_{AB}}$	**0**	**0**	**0**	**0**	**0**

CONCLUSIONS AND FUTURE WORK

The algorithm for synthesis of generalized mutually orthogonal complementary signals, suggested in the paper, generalizes the classical result of Tseng and Liu (Theorem 11 in [4]). It has been implemented in a computer program for design of *PM* signals with arbitrary phase manipulation, which provide very high level of the noise immunity, effective usage of the electromagnetic spectrum and electromagnetic compatibility of different devices. The obtained results demonstrate that the proposed algorithm could be useful in the process of computer design of perspective radars, radio-navigation, synchronization and *MC-DS-CDMA* communication systems.

REFERENCES

[1] B. Y. Bedzhev, Z. N. Tasheva, B. P. Stoyanov, "An algorithm for computer synthesis of complemntary sequences, Proceedings of the International Conference on Computer Systems and Technologies CompSysTech'04, 17-18 June 2004, Rousse, Bulgaria, pp. III B.11.1- III B.11.6

[2] H.-H. Chen, S. Yat-Sen S., J.-F. Yeh, N. Suehiro, "A multi-carrier CDMA architecture based on orthogonal complementary codes for new generations wideband wireless communications", IEEE Communication Magazine, 2001, vol. 39, № 10, pp. 126 – 135

[3] M. J. E. Golay, "Complementary Series", IRE Transaction on Information Theory, 1961, vol. IT-7, № 2 (April), pp. 82 – 87

[4] C.-C. Tseng, C. L. Liu, "Complementary Sets of Sequences", IEEE Transaction on Information Theory, 1972, vol. IT-18, № 5 (September), pp. 644 – 652

[5] Варакин Л. Е., Системы связи с шумоподобными сигналами - М.: Радио и связь,1985. – 384 с.

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