Bayesian Network Learning for Rare Events

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Abstract: *Parameter learning from data in Bayesian networks is a straightforward task. The average number of observed occurrences is stored in a conditional probability table, from which future predictions can be calculated. This method relies heavily on the quality of the data. A data set with 'rare events' will not yield statistically reliable estimates. Bayesian networks allow prior and posterior learning. In this paper, new prior assessment techniques are introduced to obtain stable priors for a conditional probability table.*
These learning algorithms are implemented and tested, and the results will be presented.

Key words: *Bayesian networks, Learning, Naive Bayes priors.*

INTRODUCTION

A Bayesian network [8, 10] is a directed acyclic graph with each node representing a variable and each arc representing a causal relation between two variables. Variables are characterized by a probability distribution for each value. The probability distribution of each node is influenced by the states (for discrete nodes) or values (for continuous nodes) of the parent node. The conditional probabilities of a node are stored in a conditional probability table (CPT). The CPT is needed to calculate any conditional probability in the model, inference [5]. The size of the CPT depends on the number of states (s), the number of parents (p), and the number of parent states (sp) in the following way:

$$size(CPT) = s \cdot (s_p)^p \qquad (1)$$

For every possible combination of parent states, there is an entry listed in the CPT. Notice that for a large number of parents the CPT will expand drastically. Assume the variables in the Bayesian network illustrated in Figure 1 are binary.

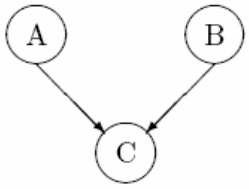

Figure 1: Example Bayesian network

The conditional probability table of node C will have eight entries, with four degrees of freedom. The CPT for node C is given in Table 1.

Table 1: CPT for node C

$p(C \mid AB)$	c	$\bar{c}$
a,b	θ_{ab}	$1-\theta_{ab}$
$a,\bar{b}$	$\theta_{a\bar{b}}$	$1-\theta_{a\bar{b}}$
$\bar{a},b$	$\theta_{\bar{a}b}$	$1-\theta_{\bar{a}b}$
$\bar{a},\bar{b}$	$\theta_{\bar{a}\bar{b}}$	$1-\theta_{\bar{a}\bar{b}}$

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The number of degrees of freedom of a CPT with number of states (s), number of parents (p), and number of parent states (s_p) is $s_p{}^p(s-1)$. Values for θ can be obtained from the data by parameter learning. θ_{ab} is the number of occurrences (c,ab) divided by the number of occurrences (ab), or (c,ab) + $(\bar{c},ab)$. In general, if for a variable X, with states $x_1,...,x_N$, p_i is the i-*th* combination of parent states, then θ_{ij} denotes the probability of x_j given p_i. Let $o(p_i,x_j)$ be the number of observations (p_i,x_j) in the data set. θ_{ij} can now be calculated as follows:

$$\theta_{ij} = \frac{o(p_i,x_j)}{\sum_{k=1}^{N} o(p_i,x_k)} \quad (2)$$

Parameter learning is basically counting the number of observations of a specific event. Notice that the accuracy of θ heavily depends on the number of observed events. A large number of observations will result in a more accurate estimate than just a small sample. Therefore, large data sets usually provide good, stable models. Unfortunately many real-world data sets are imbalanced; some states of the response variable may be dozens to thousands of times less likely than other states. This is the case in, for example, customer bankruptcies in banks, international armed conflicts, or epidemiological infections. Thus the effect of having a large data set available is canceled by the fact that it contains only a small number of 'interesting' records. Therefore, a model based on such data may be severely biased or highly unstable.

PREVIOUS WORK

Rare event problems occur in generalized linear models, such as logistic regression, but also in models learned from data, such as neural networks and Bayesian networks. Especially in the case of generalized linear models, techniques such as prior correction and weighting have been developed to discard most of the 'uninteresting' part of the data set without much performance loss [4]. Additionally there are ways of reducing bias and variance [9]. In Bayesian networks, small sample problems are usually solved by setting a good prior distribution [1, 6], based on expert knowledge. However, if this expert knowledge is not available, an acceptable prior, based on the data, needs to be set. [7] proposes noisy-OR to obtain priors. This approach uses cutting to shift from prior to posterior estimate. Naive Bayes is a very good classifier, as described in [2]. Parameter learning is explained in detail by David Heckerman in [3].
Inference in a Bayesian network is the calculation of conditional probabilities, given the probabilities in a CPT. Inference is based on two rules. The first one is Bayes' rule, which is defined as:

$$p(x \mid y) = \frac{p(y \mid x)\,p(x)}{p(y)} \quad (3)$$

The second rule is the expansion rule, which is defined for binary variables ($p(\bar{x}) = 1 - p(x)$) as:

$$
\begin{aligned}
p(x) &= p(x \mid y)\,p(y) + p(x \mid \bar{y})\,p(\bar{y}) \\
p(x) &= p(x \mid yz)\,p(yz) + p(x \mid \bar{y}z)\,p(\bar{y}z) + p(x \mid y\bar{z})\,p(y\bar{z}) + p(x \mid \bar{y}\bar{z})\,p(\bar{y}\bar{z}) \\
&= \sum_{Y}\sum_{Z} p(x \mid yz)\,p(yz)
\end{aligned}
\qquad (4)
$$

Using these two rules, in the example in Figure 1, $p(a \mid c)$ can now be calculated:

$$
\begin{aligned}
p(a \mid c) &= \frac{p(a)\,p(c \mid a)}{p(c)} \\
&= \frac{p(a)\sum_{B} \mathrm{p(c \mid ab)p(b)}}{\sum_{\mathrm{A}}\sum_{B} \mathrm{p(c \mid ab)p(ab)}}
\end{aligned}
\qquad (5)
$$

Inference will be used for the assessment of naive Bayes priors.

CPT Calculation

Deriving CPT parameters using a prior-posterior approach consists of three stages:

1. prior assessment
2. posterior assessment
3. merging

The posterior assessment is equivalent with CPT parameter learning, shown in equation 2, and will not be discussed here. First, the merging method is described and then the prior assessment.

Merging

The merging process of priors and posteriors can be handled in a couple of ways. [7] uses a cut value for the number of observations, in that paper called smoothing. If, in the data set, a combination of parent states pi occurs less than the cut value, the prior will be used in the CPT. Otherwise, the posterior will be used. Let the prior for θ_{ij} be p_{ij} , the posterior r_{ij} , and the cut value c, the smoothing merging is defined as:

$$
\theta_{ij} = \begin{cases} p_{ij}, if \sum_{\mathrm{k=1}}^{\mathrm{N}} \mathrm{o(p_i, x_k)} < \mathrm{c} \\ r_{ij}, otherwise \end{cases}
\qquad (6)
$$

Substitution of equations 2 and 6 yields:

$$
\theta_{ij} = \begin{cases} p_{ij}, if \sum_{\mathrm{k=1}}^{\mathrm{j}} \mathrm{o(p_i, x_k)} < \mathrm{c} \\ \dfrac{\mathrm{o(p_i, x_j)}}{\sum_{k=1}^{N} \mathrm{o(p_i, x_k)}}, otherwise \end{cases}
\qquad (7)
$$

In this paper, a gentle transition is proposed, by weighting the priors. The priors will receive an integer value, weight w, which is equivalent to a number of observations. The resulting CPT entries will be a weighted average of the priors and the posteriors:

$$\theta_{ij} = \frac{(w \cdot p_{ij}) + o(p_i, x_j)}{w + \sum_{k=1}^{j} o(p_i, x_k)} \qquad (8)$$

If $o(p_i, x_i)$ is low, the CPT value will mainly be based on the prior, however, if (p_i, x_i) is often observed in the data set, the data will influence the CPT value more than the prior. Basically this effect is the same as smoothing, only the transition is gentler. In smoothing and weighting, the values for c and w, respectively, need to be set. The tests in Section 4 show that these parameters have the greatest effect if a value between 5 and 80 is chosen, preferably between 10 and 40. If the values are too low, the procedure will be similar to normal CPT learning. If the values are too high, the posteriors will have very little effect. Notice that for a model with extremely good priors, the weight or cut value should be very high.

Prior Assessment

A CPT contains very detailed information about conditional probabilities for all possible parent states. As shown in the introduction, the number of degrees of freedom expands rapidly if the number of parents and the number of parent states increase. A stable prior should therefore be derived from a low number of degrees of freedom, but as accurate as possible. [7] propose the usage of noisy-MAX priors, because it is a good modeling technique for rare events. Noisy-MAX makes the assumption that the state of the response variable is a logical combination of the states of the input parameters. In practice, for a lot of data sets, noisy-MAX shows high performance, because in many cases, an effect is an addition or multiplication of the causes. An example of a binary noisy-MAX (noisy-OR) model is given in Figure 2.

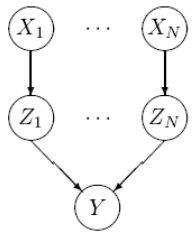

Figure 2: Noisy-MAX network

Aside from the input variables $X_1 \ldots X_N$ and response variable Y, there is a set of inhibitor nodes $Z_1 \ldots Z_N$. X_i can cause Z_i to be present with a probability c_i, but absence of X_i always implies absence of Z_i. The CPT of node Y is similar to a logical MAX gate. Naive Bayes is in many cases a superior classifier [2]. Just as noisy-MAX, naive Bayes has a relatively low number of degrees of freedom. The performance as a classifier is slightly better than noisy-MAX. The network structure is completely opposite from a regular CPT network. It may be against intuition to use priors from an opposite network. In the prior assessment, the causalities are less relevant than how $p(y \mid p_i)$ is calculated. A naive Bayes network makes very unrealistic assumptions about causality. It assumes the predictive variables to be dependent on the response variable. Also, it assumes conditional independence between the predictive variables, meaning that given the value of Y, $X_1 \ldots X_N$ are independent. Aside from these assumptions, a naive Bayes model is a

very powerful classifier. Inference in a naive Bayes network is not as straightforward as in a CPT network. A network with predictive variables $X_1 \ldots X_N$ and response variable Y with states $y_1 \ldots y_M$ will have the probabilities $p(y \mid p_i)$ listed in the CPT. In a naive Bayes network, these values need to be calculated as follows:

$$\begin{aligned} p(y \mid p_i) &= \frac{p(p_i \mid y)\,p(y)}{p(p_i)} \\ &= \frac{\prod_i p(x_i \mid y)\,p(y)}{\prod_i p(x_i)} \\ &= \frac{\prod_i p(x_i \mid y)\,p(y)}{\prod_i \sum_j (p(x_i \mid y_j)\,p(y_j))} \end{aligned} \qquad (9)$$

The values for $p(x_i \mid y)$ and $p(y)$ can be obtained by parameter learning in the naive Bayes network. p_{ij} from equation 8 can be replaced by the right part of equation 9. Now the formula for calculation of the CPT values is complete.

RESULTS

The method above was implemented and tested on a bankruptcy data set from a bank. The bankruptcies were rare (around 2% of all records). As a performance measure for scoring the models, the GINI index was used. In a graph where a curve is plotted as the cumulative bankrupt percentage of clients against the total percentage of clients when they are sorted on risk (low risk on the left, high risk on the right), the GINI index is defined as the area between the diagonal and the curve (the Lorentz curve) divided by the total area under the diagonal. It has a range between -1 and 1, or −100% to 100%. High scores indicate good models. The GINI index is widely used in social sciences to measure the discriminative power of a model.

Table 2: Performance of models

MODEL	GINI
Naive Weighting W=1	76.4%
Naive Weighting W=5	77.0%
Naive Weighting W=10	77.1%
Naive Weighting W=20	77.2%
Naive Weighting W=40	77.3%
Naive Weighting W=80	77.3%
Noisy-MAX Weighting W=10	71.4%
Naive Smoothing C=10	75.4%
Naive Smoothing C=20	76.6%
Naive Smoothing C=40	77.0%
Noisy-MAX Smoothing C=20	71.4%

More information about the GINI index and its applications can be found in [11]. A couple of modeling methods were compared, of which the results are listed in Table 2. Apparently, for this data set, naive Bayes provides much better priors than noisy-MAX. This is partly due to the fact that a suboptimal learning algorithm for noisy-MAX is used.

Even with EM learning, noisy-MAX scores will not be higher than 76%. Secondly, the weighting seems to perform better than the smoothing for this data set.

CONCLUSIONS AND FUTURE WORK

Two improvements to learning for rare event data were suggested in this paper. Firstly, weighted merging instead of cutting, which allows a more gentle balance between a prior and a posterior. Secondly, inference in a naive Bayes models can provide excellent priors for a CPT in a 'normal' Bayesian network.
Compared to existing methods, such as noisy-MAX priors, naive Bayes priors perform better on the test data set. Unfortunately, the best performing model on this data set was naive Bayes without any posterior learning. Therefore, more data sets to test these methods on are required for a definite statement. These initial results are very promising.

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