Generator of fractional Gaussian noise for modelling self-similar network traffic

Mitko Gospodinov, Evgeniya Gospodinova

Abstract: *Recent network studies of high-speed networks show that commonly used teletraffic models, based on Poisson or related processes are nor able to capture the self-similar nature of teletraffic. The present paper focuses on self-similar traffic generation, based of fractional Gausssian noise (FGN) using a linear approximation. This linear approximation reduces the complexity of the computation without compromising the accuracy in synthesizing the power spectrum of the FGN.*

Key words: *fractional Gaussian noise (FGN), self-similar network traffic.*

1. Introduction

The self-similar processes are with long-range dependence, and they are characterized by hyperbolically decaying autocorrelation functions [2]. The self-similar traffic models are attractive network models because the long-range dependence is characterized by single *Hurst (H)* parameter and they are much better than Poisson models [3].

There are three mathematical models used for generating self-similar network traffic: The fractional Gaussian noise (FGN), the fractional Brownian motion (FBM) and the fractional autoregressive integrated moving average (F-ARIMA) process [1]. In this paper is investigated the use of FGN for generating self-similar network traffic. This method generates self-similar traffic with high degree of accuracy with given Hurst parameters.

2. The fractional Gaussian noise

The numbers representing the spectral density function of FGN is obtained by applying appropriate transformations to originally uniformly distributed pseudo-random numbers. The power spectrum of FGN is formulated by [2]:

$$F(\lambda,H)=A(\lambda,H)[\lambda^{-2H-1}+B(\lambda,H)] \quad \text{for } 0<H<1 \text{ and } -\pi \le\lambda\le\pi \tag{1}$$

Where:

$$A(\lambda,H)=2\sin(\pi H)\Gamma(2H+1)(1-\cos \lambda)$$

$$B(\lambda;H) = \sum_{k=1}^{\infty} [(2\pi k + \lambda)^{-2H-1} + (2\pi k - \lambda)^{-2H-1}] \tag{2}$$

In the above expression for $A(\lambda,H)$, $\Gamma(.)$ indicates the Gamma function. The main difficulty of using equation (2) to obtain the samples of the power spectrum is the infinite number of summation in $B(\lambda;H)$. The equation (2) can re-write as:

$$B(\lambda;H) = \sum_{k=1}^{2} [(2\pi k + \lambda)^{-2H-1} + (2\pi k - \lambda)^{-2H-1}] + B_{3:\infty} \tag{3}$$

Where:

$$B_{3:\infty} = \sum_{k=3}^{\infty} [(2\pi k + \lambda)^{-2H-1} + (2\pi k - \lambda)^{-2H-1}] \tag{4}$$

Using a linear function $D(\lambda;H)=p\lambda+q$ to approximate $B_{3:\infty}$, and determining the optimal p and q according to the mean-squared error criterion is obtained:

$$\begin{cases} p = -\dfrac{6}{\pi^2}F(H) + \dfrac{12}{\pi^3}G(H) \\ q = \dfrac{4}{\pi}F(H) - \dfrac{6}{\pi^2}G(H) \end{cases} \tag{5}$$

Where:

$$F(H) = \sum_{k=3}^{\infty}\left[\frac{(2\pi k-\pi)^{-2H}-(2\pi k+\pi)^{-2H}}{2H}\right] \quad (6)$$

and

$$G(H)=\sum_{k=3}^{\infty}[((2\pi k)(2\pi k+\pi)^{-2H}+(2\pi k)(2\pi k-\pi)^{-2H}-2(2\pi k)^{-2H+1})/2H - ((2\pi k+\pi)^{-2H+1}+(2\pi k-\pi)^{-2H+1}-2(2\pi k)^{-2H+1})/(2H-1)] \quad (7)$$

The computation of equations (1) and (2) for the power spectrum of the FGN, infinite summations are required for every value of frequency. The linear approximation is used in equation (3) for computation of $B_{3:\infty}$.

3. Simulation results

The main goal of this paper is created a generator of pseudo-random self-similar sequences, based on the fractional Gaussian noise using a linear approximation. The generator is implemented in C++ language.

The calculation of the power spectrum of the FGN implies the computation of an infinite summation. To fine accurate way to approximate equation (2) are analysed the terms in the summation. Each term is a function of the index of the summation (k) and the frequency (λ). The first term $(2\pi k + \lambda)^{-2H-1}$ decreases as k goes to infinity. For large values of k this term remains approximately constant for all the values of λ. Table 1 shows the relative errors between $(2\pi k + \lambda)^{-2H-1}$ and its linear approximation for several values of k, λ and $H=0.7$. The values of p and q in equation 5 are determined by minimizing a mean-squared error. For k≥3 the deviation of $(2\pi k + \lambda)^{-2H-1}$ is very small.

Table 1

λ	k=1	k=2	k=3	k=4
0,314	6,42%	2,19%	1,10%	0,66%
0,628	1,67%	0,65%	0,34%	0,21%
0,942	-1,84%	-0,51%	-0,23%	-0,13%
1,257	-4,18%	-1,29%	-0,62%	-0,36%
1,571	-5,36%	-1,69%	-0,82%	-0,48%
1,885	-5,32%	-1,68%	-0,81%	-0,48%
2,199	-3,90%	-1,24%	-0,6%	-0,36%
2,513	-0,80%	-0,33%	-0,17%	-0,11%
2,827	4,50%	1,11%	0,49%	0,28%
3,142	12,96%	3,16%	1,41%	0,80%

Table 2 shows the relative errors between the second term in equation (2) and its linear approximation. The conclusion is similar to the above.

For evaluation of equation (2) is used partial summation for $k=1, 2,\ldots, n$. The first few terms are big compared to the terms for large value of k. This is illustrated in Figure 1. The plots were obtained using $\lambda= \pi/2$. The results are very similar for other values of λ. The summation B_2 for $k=1, 2$ and $H=0.9$ represent more than *93%* of B. The summation B_2 for $H=0.6$ is *85%* of B.

Table 2

λ	k=1	k=2	k=3	k=4
0,314	96,81%	6,02%	2,11%	1,07%
0,628	24,52%	2,12%	0,74%	0,37%
0,942	-0,75%	-0,52%	-0,24%	-0,14%
1,257	-11,77%	-2,15%	-0,87%	-0,47%
1,571	-16,13%	-2,92%	-1,17%	-0,63%
1,885	-16,32%	-2,94%	-1,18%	-0,63%
2,199	-13,21%	-2,26%	-0,9%	-0,48%
2,513	-6,86%	-0,93%	-0,35%	-0,18%
2,827	3,23%	1,04%	0,47%	0,27%
3,142	18,2%	3,67%	1,56%	0,86%

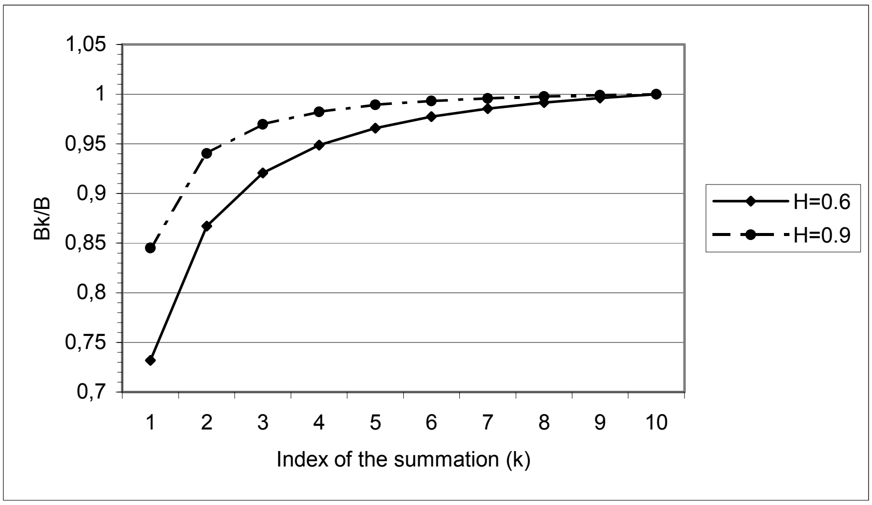

Figure 1. Dependence of B_k/B on k for $H=0.6$ and $H=0.9$

The illustration in Table 1 and Table 2 suggest that a better approximation using a linear function of B in equation (2) should not include the terms for k=1 and k=2. For any H, $B_{3:\infty}$ can be perfectly approximated, using a linear function of λ, as illustrated in Figure 2.

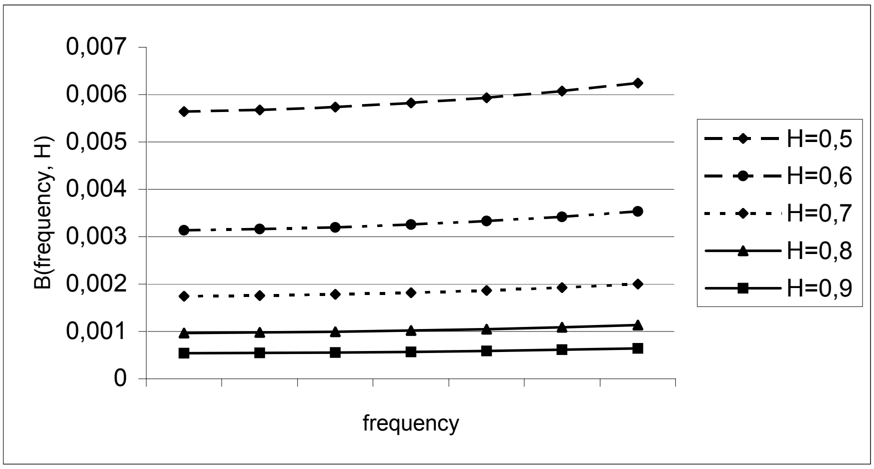

Figure 2. Dependence of $B_{3:\infty}$ on frequency for different H values

For the computation of parameters *p* and *n* are performed two infinite summations in equations (6) and (7). Figures 3 and 4 show plots of the functions P_n and Q_n versus the number of terms used in summation (n) for several values of *H.*

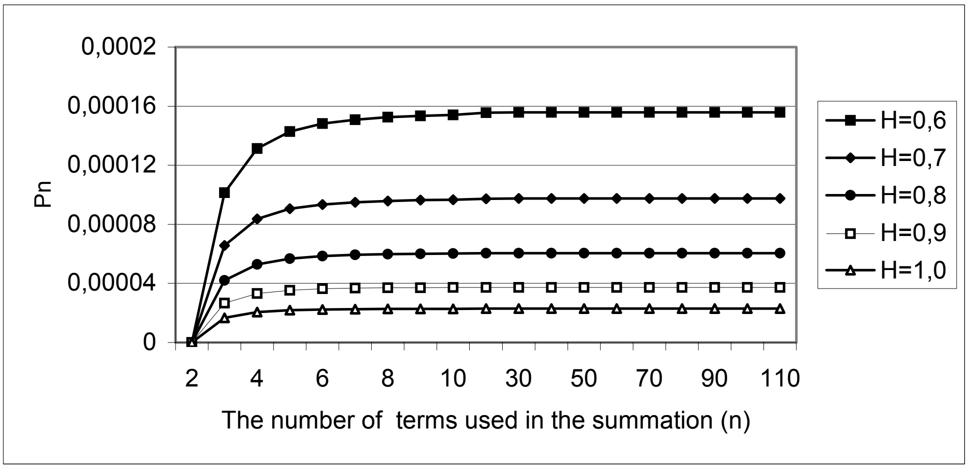

Figure 3. Dependence of *Pn* on *n* for different *H* values

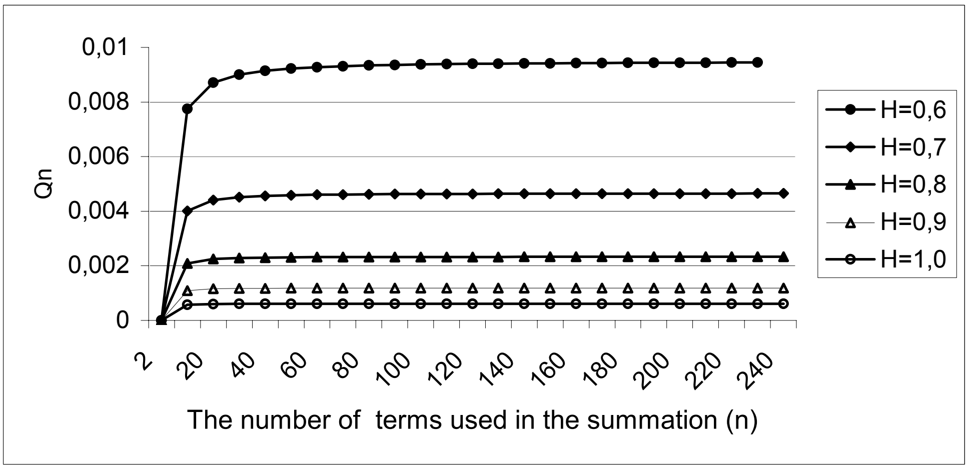

Figure 4. Dependence of *Qn* on *n* for different *H* values

As illustrated in Figure 3 , the slope of the linear approximation *P* converges very fast and a value of n=8 seems to be sufficient. The parameter *Q* in Figure 4 does not converge that fast, a minimum value of n=30 are required.

The relative error for the computation of *B(λ,H)* is defined as:

$$Error = \frac{B(\lambda,H) - [B_{1:2}(\lambda,H) + p\lambda + q]}{B(\lambda,H)}$$
$$= \frac{B_{3:\infty}(\lambda,H) - (p\lambda + q)}{B(\lambda,H)} \quad (8)$$

For the calculation of *B(λ,H)* in (8) is used 10000 terms. The Figure 5 shows the relative error for several values of *H.* The relative error does not exhibit any bias and it is almost independent of *H* with a maximum value of 0.15%.

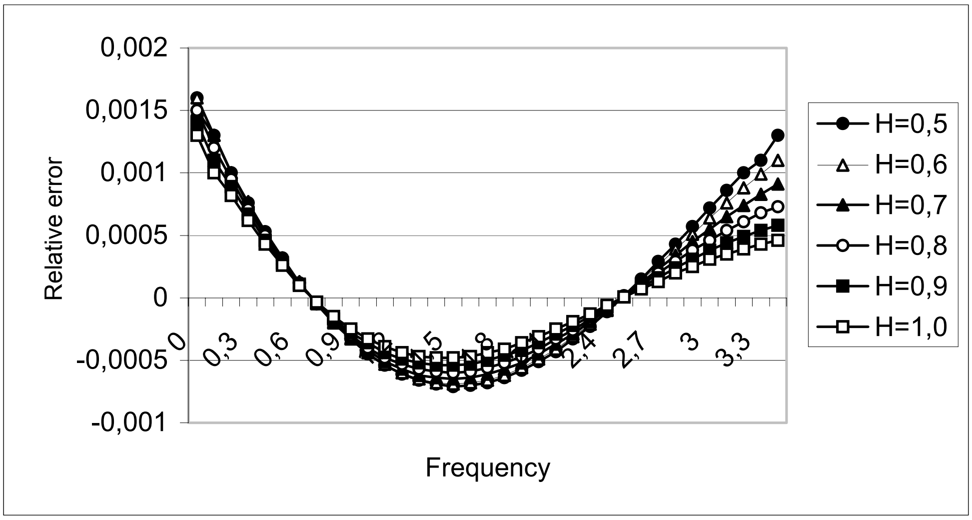

Figure 5. Relative error for several values of *H*

4. Conclusions

In this paper is presented a generator of pseudo-random self-similar sequences, based on the FGN using linear approximation. To deal with the infinite summation of the FGN spectral density function are computed the first two terms exactly and approximated the rest of the summation using a linear function. The simulation results show that the guarantees to generate self-similar network traffic are with high degree of accuracy.

This generator of self-similar sequences can be used as a source model for the simulating network traffic including: Ethernet, ATM, VBR coded video, Web traffic, Telnet and FTP.

References

[1] Beran, J., R. Sherman. "Long-range dependence in variable-bit-rate video traffic". IEEE Transactions on Communications, vol.43, 1995.

[2] Leland, W., M.Taqqu. "On the self-similar nature of Ethernet traffic". IEEE/ACM Transactions on Networking, vol.47, 1993.

[3]Willinger, W., M.S.Taqqu. "Self-similarity through high-variability:Statistical analysis of Ethernet LAN traffic at the source level". IEEE/ACM Transactions on Networking, vol.5, 1993.

ABOUT THE AUTHOR

Assoc. Prof. Mitko Gospodinov, PhD, Central Laboratory of Mechatronics and Instrumentation at Bulgarian Academy of Sciences, Fhone: +359 887 426 490, E-mail: mitgo@abv.bg

Assistant Prof. Evgeniya Peneva Gospodinova, Central Laboratory of Mechatronics and Instrumentation at Bulgarian Academy of Sciences, Phone: +359 886 626 911, E-mail: jenigospodinova@abv.bg