An Algorithm for Synthesis of Aperiodic Optical Orthogonal Codes

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Abstract: *Communication and computer systems have formed a common space where the quantity and quality of offered services are growing very fast at present. This situation leads to necessity of extreme optimal using of the connecting channels. A promising approach to the solution to this hard technical problem is applying of optic fibre systems with Code Division Multiple Access. The employing a special class of codes, named Optical Orthogonal Codes, enables to simplify greatly the complexity of the optic fibre system, to implement it with available technology, and to achieve potentially higher transmission efficiency. Unfortunately some important problems in the design of these signals are still open. Due to this reason our paper aims to suggest a computational algorithm for synthesis of Aperiodic Optical Orthogonal Codes.*

Key words: *Computer Systems and Technologies, Computational Algorithm for Synthesis of Aperiodic Optical Orthogonal Codes.*

INTRODUCTION

The communication and computer systems are in an unprecedented progress today. These areas are closely connected and every technical innovation in each of them leads to a considerable advance in the other. Indeed communication and computer systems have formed a common space where the quantity and quality of offered services are growing very fast at present. This situation leads to necessity of extreme optimal using of the connecting channels. A promising approach to the solution to this hard technical problem is applying of optic fibre systems with Code Division Multiple Access (CDMA). The most important positive features of CDMA systems are: simplicity and flexibility, opportunity very large number of asynchronous users to share a common optic fibre channel and to transmit information efficiently with both high information rate and great reliability [1]-[3]. Traditional multiple-access approaches such as frequency division, time division, collision detection, or demand assignment require elaborate network synchronization at high speed (often optical speed), and frequent conversions between the optical domain and the electronic domain. These requirements limit the efficiency of every optical multiple-access system. However, by employing a code-division multiple-access system with a special class of codes, named Optical Orthogonal Codes (OOC), we are able to simplify greatly the complexity of the system, to implement it with available technology, and to achieve potentially higher transmission efficiency [1]-[3].

By definition [1]-[3], an OOC is a family of sequences, comprising only 0's and 1's, with good auto- and cross-correlation properties, i.e., the autocorrelation function (ACF) of each sequence exhibits the "thumbtack" shape and the cross-correlation function (CCF) between any two sequences remains low throughout.

Due to their positive features, the OOCs have been studied intensively during the past twenty years, but the most papers are focused on the synthesis of OOCs with good periodical correlation properties. Nevertheless the aperiodic correlation properties might be more appropriate for the present application. With regard, our paper aims to suggest a computational algorithm for synthesis of aperiodic (or impulse) optical orthogonal codes.

The paper is organized as follows. First, the basics of the aperiodic OOCs (AOOCs) are recalled. After then, a computational algorithm for synthesis of AOOCs is suggested. Finally, the advantages and possible areas of application of our method are discussed.

AN ALGORITHM FOR SYNTHESIS OF APERIODIC OPTICAL ORTHOGONAL CODES

In the beginning of this section, the definition and some fundamental properties of AOOCs will be given.

The most important peculiarity of the OOCs is that they consist of truly (***0, 1***) sequences and are intended for "unipolar" environments that have no negative components. In contrast most documented sequences, possessing good correlation properties, are actually (***+ 1, - 1***) sequences intended for systems having both positive and negative components. This important distinction produces quite different results [1]-[3]. As aforementioned, our paper is focused on synthesis of impulse OOCs (i.e OOCs with good aperiodic correlation properties). Due to this reason, we shall use the following definition.

Definition: An $(n,w,\lambda_a,\lambda_c)$ aperiodic optical orthogonal code ***C*** is a family of (***0, 1***) sequences of length ***n*** and weight ***w*** (number of ***1***s $w \le n$) which satisfy the following two properties.

$$R_{kk}(r) = \sum_{i=1}^{n-1-r} \xi_k(i).\xi_k(i+r) \le \lambda_a,\ r \ne 0\,, \tag{1}$$

$$R_{ks}(r) = \sum_{i=1}^{n-1-r} \xi_k(i).\xi_s(i+r) \le \lambda_c\,, \tag{2}$$

where:

- $R_{kk}(k)$ is the aperiodic ACF of the ***k***-th sequence $\{\xi_k(j)\}_{j=0}^{n-1}$ belonging to the family ***C***;
- $R_{ks}(k)$ denotes the periodic CCF between ***k***-th and ***s***-th sequences of the family ***C***;
- $r = 0, 1, 2, ...n-1$ is the horizontal (time) shift;
- the positive integers λ_a, λ_c are the maximum permissible levels (thresholds, constrains) of ACFs and CCFs in the family ***C*** respectively.

The (***0, 1***) sequences of an optical orthogonal code are often called its codewords. The size of an optical orthogonal code, denoted by $|C|$, is the number of codewords in it [1]-[3]. Throughout this paper in order to avoid triviality it will be required:

$$\lambda_a, \lambda_c \le w\,. \tag{3}$$

From definition it follows that the ACF of each sequence will exhibit a "thumbtack" shape and the CCF between any two sequences will remain low throughout if the positive integers λ_a, λ_c are small comparatively to ***w***.

The notations of the AOOCs will be clarified by following Example 1.

Example 1: $C = \{(1100\ 1000\ 0000), (1010\ 0001\ 0000)\}$ is an AOOC with parameters $n = 13, w = 3, \lambda_a = 1, \lambda_c = 1$ and containing two codewords $CW_1 = (1100\ 1000\ 0000)$, $CW_2 = (1010\ 0001\ 0000)$. Often a shorthand notation named "set notation" or "pattern notation" $C = \{(0,1,4), (0,2,7)\}$ is used. Here $\eta_0 = 0, \eta_1 = 1, \eta_2 = 2, \eta_3 = 4, \eta_4 = 7$ are the positions of ***1***s.

Now the application of AOOCs will be explained in more details [2], [3].

Let an $(n,w,\lambda_a,\lambda_c)$ aperiodic optical orthogonal code ***C*** with ***M*** codewords be utilized in an optical communication system in which ***M*** users are connected by a single optic fibre channel. In other words, the system accommodates ***M*** transmitters simultaneously. Each transmitter is assigned a sequence from ***C***. At a transmitter every information bit is encoded into a frame of ***n*** optical chips comprising ***w*** photon pulses. This is clarified on Fig. 1a, where a codeword of an AOOC with $n = 7, w = 4$ is shown. Every chip is an optical time slot with duration τ_p which can assume one of two values: ***ON*** (if there is a photon pulse) or ***OFF*** (otherwise). Let the assigned sequence for a particular transmitter be $\{\xi_k(j)\}_{j=0}^{n-1}$ and let it has ***1***s at positions $\eta_0, \eta_1, ..., \eta_{w-1}$. Assume the information bit is ***1***. In the corresponding frame, which consists of ***n*** optical chips, photon pulses (i.e., ***ON*** signals) are sent at moments $t_0 = \eta_0.\tau_p, t_1 = \eta_1.\tau_p, ..., t_{w-1} = \eta_{w-1}.\tau_p$, i.e. exactly at the $\eta_0 - th$, $\eta_1 - th$, …., and $\eta_{w-1} - th$ chips. In the other ***n - w*** chips, no photon pulses (i.e., ***OFF*** signals) are sent. Hence, the codeword is used as a signature sequence of the transmitter. On the other hand, if the information bit is ***0***, no photon pulses are sent in the corresponding frame, i.e., only ***OFF*** signals are sent. All ***M*** users are allowed to transmit at any time frame – i.e. there is required a simple network synchronization. At the receivers, correlation-type decoders are used to separate the transmitted signals. Each decoder consists of a bank of ***M*** tapped delay-lines, one for each codeword. The delay taps on a particular line exactly match the signature sequence, i.e. the delays between successive taps are equal to:

$$\mu_i = \frac{\tau_i}{\tau_p}, \; i = w-1, w-2, ..., 2, 1, \tag{3}$$

optical chips, respectively. Here $\mu_{w-1}, \mu_{w-2}, ..., \mu_2, \mu_1$ are the relative time-gaps between photon pulses in the signature sequence, arranged in reversal order. As a result each tapped delay-line effectively calculates the correlation of the received waveform with its signature sequence (in the considered case it is $\{\xi_k(j)\}_{j=0}^{n-1}$). According to the properties of AOOCs (see Eqs. (1), (2)), the correlation between different signature sequences is low. Thus the delay-line output is high only when the intended transmitter's information bit is ***1***. The transmitted information is extracted by thresholding the correlator output. This is clarified on Fig. 1b, where the ACF of the codeword from Fig. 1a is depicted. As shown, in the moment of arriving of the last pulse the ACF has an absolute maximum which exceeds the threshold and indicates that the received bit is ***1***. In general, evaluating of the ACF of the expected sequence maximizes the signal-to-noise ratio in receiver output in the presence of Gaussian noise (i. e. maximizes the probability of right discovering of a sent bit).

The above described optical CDMA system can be easily implemented. The tapped delay-line correlator is readily available. Little or no electronic-optical domain conversion is required. There is no synchronization requirement in the network. Although bandwidth expansion is affected by the transmitter, the simplicity and flexibility of the system concept enables to pump optical pulses at a much faster chip rate than otherwise possible.

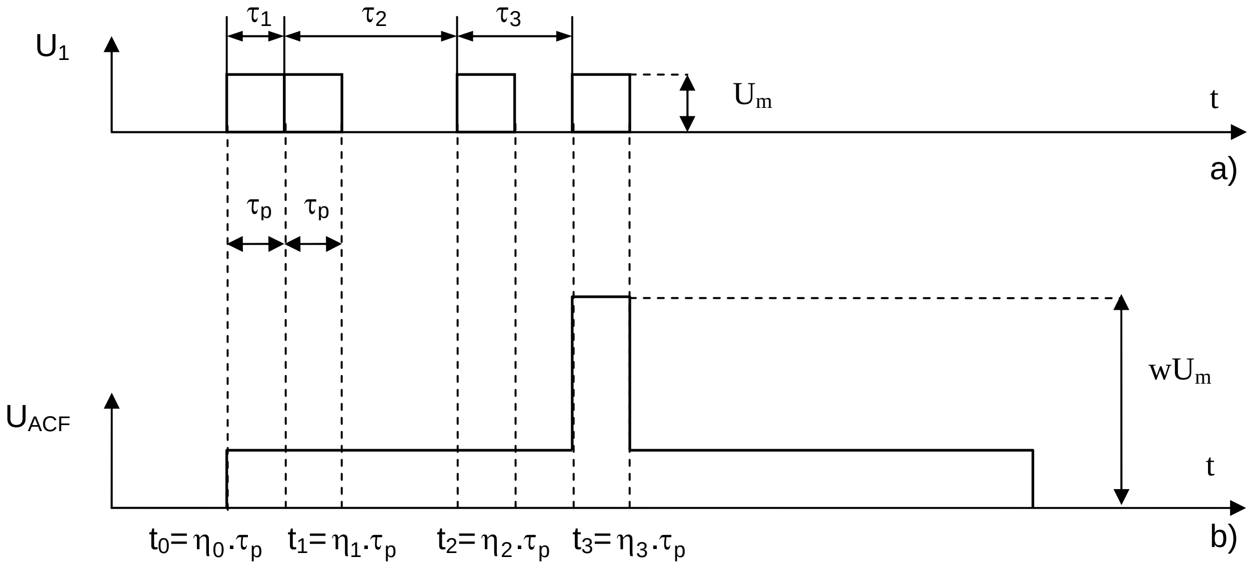

Fig.1: *Example of a codeword of an AOOC (**n**=7, **w**=4)*

From all above stated the following conclusions can be made:

1) The overall system throughput efficiency can be much improved if the optic fibre communication system utilizes OOCs as signature sequences.

2) It is desirable to have maximal energy-to-length ratio $\frac{w}{n}$ for given ***n*** and a great number of codewords in the OOC. For a given set of values of $n, w, \lambda_a, \lambda_c$, the largest possible size of an $(n, w, \lambda_a, \lambda_c)$ optical orthogonal code is denoted by $\Phi(n, w, \lambda_a, \lambda_c)$. An optical orthogonal code having both maximal energy-to-length and maximum size will be named *optimal* in this paper. Both the determination of the exact values of $\Phi(n, w, \lambda_a, \lambda_c)$ and the specific construction of optimal codes have great theoretical and practical importance.

With regard to these conclusions, in the next part of the paper a computational algorithm for synthesis of AOOCs with parameters $\lambda_a = \lambda_c = 1$ will be suggested. It should be mentioned that the case $\lambda_a = \lambda_c = 1$ has the greatest practical importance because it provides the clearest contrast between ***1***s and ***0***s in the output of the receiver.

First of all the following Proposition 1, giving an important necessary condition a sequence to be a codeword of an $\lambda_a = \lambda_c = 1$ AOOC, will be proved.

Proposition 1: In every codeword of an AOOC all successive sums:

$$S(k,m) = \sum_{i=k}^{m} \mu_i, \; k = 1,2,...w-1, \; m = k, k+1, ..., w-1 \tag{4}$$

are different.

Proof: The proposition will be proved by means of so-named "method of formal polynomials (or formal power sums of a single variable)" [4]. According to this method, a codeword consisting of ***w*** elementary unity pulses, put in the relative time-positions:

$$\eta_0 = 0; \eta_1 = \mu_1; \eta_2 = \mu_1 + \mu_2; \eta_3 = \mu_1 + \mu_2 + \mu_3, ...\eta_{w-1} = \mu_1 + \mu_2 + ... + \mu_{w-1}, \tag{5}$$

is presented by the polynomial:

$$F(x) = x^{\eta_{w-1}} + x^{\eta_{w-2}} + ... + x^{\eta_2} + x^{\eta_1} + 1. \tag{6}$$

Then the lobes of the ACF of the codeword are the coefficients of the polynomial:

$$P(x) = F(x).F(x^{-1}), \tag{7}$$

where:

$$F(x) = x^{-\eta_{w-1}} + x^{-\eta_{w-2}} + ... + x^{-\eta_2} + x^{-\eta_1} + 1. \tag{8}$$

After evaluation of Eq. (7) the result is:

$$P(x) = \sum_{k=1}^{w-1}\sum_{m=k}^{w-1} x^{-S(k,m)} + w + \sum_{k=1}^{w-1}\sum_{m=k}^{w-1} x^{S(k,m)} . \tag{9}$$

The coefficients of the Eq. (9) right side, presenting the ACF side-lobes of the considered codeword, will be smaller or equal to $\lambda_a = 1$ only if all sums $S(k,m)$ are different. This conclusion ends the proof of Proposition 1.

The Proposition 1 shows the tight connection between the AOOC synthesis and the constructing of sequences in which all successive sums $S(k,m)$ are different. With regard we shall recall that we suggested an effective computational algorithm for solving the last problem in another our paper [5]. For convenience it will be named "Algorithm for Constructing of Sequences with Different Successive Sums" (ACSDSS). The main idea of the ACSDSS is the sequence $\{\mu_1, \mu_2, ..., \mu_{w-1}\}$ to be successively filled up by putting "step-by-step" as small as possible new values in its middle.

Our algorithm for synthesis of AOOCs is based on the following proposition.

Proposition 2: Every "Sequences with Different Successive Sums" (SDSS) can be transformed in an $\lambda_a = \lambda_c = 1$ AOOC.

Proof: Suppose, a SDSS $\{\mu_1, \mu_2, ..., \mu_{w-1}\}$ is obtained by means of the ACSDSS. Let the positions of its ***1***s be $\{\eta_0, \eta_1, \eta_2, ..., \eta_{w-1}\}$. Then the set $\{\eta_0, \eta_1, \eta_2, ..., \eta_{w-1}\}$ can be divided into ***M*** subsets:

$$CW_k = (\eta_{k0}, \eta_{k1}, ..., \eta_{kM-1});\ \forall \eta_{kj} \in \{\eta_0, \eta_1, \eta_2, ..., \eta_{w-1}\}; \quad k = 1,2,...,M;\ j = 1,2,...,v;\ v < w,\ M < w, \quad (10)$$

so that every element $\eta_j, j = 0,1,...,w-1$ belongs to maximum 2 subsets.

Let CW_k be used as signature sequences in an optical communication system in which ***M*** users are connected by a single optic fibre channel. According to above mentioned method of formal polynomials, the output reaction of the ***q***-th receiver to information sequence of the ***s***-th transmitter (if the sent bit is ***1***) is:

$$P_{sq}(x) = F_s(x).F_q(x^{-1}), \quad (11)$$

where:

$$F_s(x) = x^{\eta_{sM-1}} + x^{\eta_{sM-2}} + ... + x^{\eta_{s1}} + x^{\eta_{s0}};\ F_q(x) = x^{-\eta_{qM-1}} + x^{-\eta_{qM-2}} + ... + x^{-\eta_{q1}} + x^{-\eta_{q0}}. \quad (12)$$

From Eqs. (11), (12) it is straightforward that all coefficients of $P_{sq}(x)$, presenting the CCF side-lobes of the ***s***-th and ***q***-th codewords, are smaller or equal to $\lambda_c = 1$ because all sums $S(k,m)$ are different. Analogous arguments show that the ACF side-lobes of the ***q***-th codeword, are smaller or equal to $\lambda_a = 1$. These conclusions end the proof of Proposition 2.

Now we shall evaluate the output reaction of the ***q***-th receiver in the worst case, when all ***M-1*** users are transmitting simultaneously during an arbitrary time frame a bit ***1***, whereas the intended ***q***-th transmitter is sending a ***0***. According to Eqs. (11), (12) the total reaction of the ***q***-th receiver to information sequences of all other ***M-1*** users will be:

$$P_\Sigma(x) = \sum_{s \neq q} F_s(x).F_q(x^{-1}) = F_\Sigma(x).F_q(x^{-1}), \quad (13)$$

where:

$$F_\Sigma(x) = u_{w-1}.x^{\eta_{w-1}} + u_{w-2}x^{\eta_{w-2}} + ... + u_2 x^{\eta_2} + u_1 x^{\eta_1} + u_0, \quad (14)$$

and all coefficients are no bigger than 2, i. e. $0 \le u_j \le 2,\ j = 0,1,...,w-1$. After evaluation of Eq. (14) the result is:

$$P_\Sigma(x) = \sum_{i=1}^{\eta_{qM-1}} \sum_{j=i, j \in Q} u_i.x^{-S(i,j)} + \sum_{i \in Q} \sum_{j=i}^{w-1} u_j x^{S(i,j)}, \quad (15)$$

where $Q = \{\eta_{q0}, \eta_{q1}, ..., \eta_{qM-1}\}$ and $S(i,j)$ are successive sums ending or beginning with an element of Q depending on that if the power sign is "-" or "+".

The coefficients of the Eq. (15) right side, presenting the side-lobes of the output reaction of the ***q***-th receiver in the worst case (when all ***M-1*** users are transmitting simultaneously a bit ***1*** during an arbitrary time frame, whereas the intended ***q***-th transmitter is sending a ***0***) are smaller or equal to 2.

With regard to last conclusion and to Proposition 2, our algorithm for AOOC synthesis consists of two steps:

1) Constructing a sequence with difference successive sums $\{\mu_1, \mu_2, ..., \mu_{w-1}\}$ by means of the ACSDSS [5].

2) Dividing $\{\mu_1, \mu_2, ..., \mu_{w-1}\}$ into ***M*** codewords so that every element $\eta_j, j = 0,1,...,w-1$ is put in maximum $u_j < M, j = 0,1,...,w-1$ subsets respectively.

Example 2: The sequence (110010000010000100) (or (0,1,4,10,15,17) in set notation) is a SDSS with weight $n = 6$. The set (0,1,4,10,15,17) can be divided in 3 subsets so that every element of the set $(0,1,4,10,15,17)$ belongs to maximum 2 subsets. A possible dividing is $CW_1 = (0,1,4), CW_2 = (1,10,15), CW_3 = (4,15,17)$. Suppose these subsets are used as signature sequences in an optical fibre communication system with 3 users. The direct examination of the worst case, when ***2*** users are transmitting simultaneously a bit ***1*** during an arbitrary time frame, whereas the intended ***q***-th transmitter is sending a ***0***, shows that maximal side-lobes of the total response of ***q***-th receiver is $2U_m$. Meanwhile the main lobe of the output reaction of the ***q***-th receiver is at least $3U_m$ if the intended ***q***-th transmitter is sending a ***1***. Consequently, if the threshold is bigger than $2U_m$ (but smaller than $3U_m$!) the system will work properly even so in the worst possible practical case.

CONCLUSIONS AND FUTURE WORK

At present OOCs have applications in mobile radio, neuromorphic networks, and radar and sonar signal design. Recent work has been done on using OOCs for multimedia transmission in fibre-optic local-area networks (LANs) and in multirate fibre-optic CDMA systems, as well. Despite of proliferation of fibre optic networks the most paper, concerning the synthesis of OOCs, are focused on the periodic codes. Nevertheless the aperiodic OOCs might be more appropriate for the present application. With regard, the algorithm for OOCs, suggested above, could be very useful in the process of fibre optic communication system design. Our algorithm for AOOC synthesis was implemented in a computer program, working in Visual C++ environment. The obtained results confirmed the computational effectiveness of the algorithm.

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