An Algorithm for Synthesis of Nonsinusoidal Radar Signals

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Abstract: *The nonsinusoidal communication systems have some properties which are very suitable for a lot of application such as radars, remote synchronization systems and radio navigation. More specifically, the nonsinusoidal radars utilize very narrow radio pulses. This leads to radical improvement of the most important radar parameters as range resolution, abilities for recognizing of targets and discovering small objects on background of large obstacles. With regard to positive features of nonsinusoidal radars, they are in very rapid progress in both theoretical and practical aspects recently. Despite of taken efforts, some important problems in the design of nonsinusoidal radars are still open. Due to this reason our paper aims to suggest an effective computer algorithm for synthesis of nonsinusoidal radar signals.*

Key words: *Computer Systems and Technologies, Computer Algorithm for Synthesis of Nonsinusoidal Radar Signals.*

INTRODUCTION

The nonsinusoidal communication systems were proposed at the beginning of 70s of former century. They have some properties which are very suitable for a lot of application such as radars, remote synchronization systems and radio navigation [1], [2]. More specifically, the nonsinusoidal radars utilize very narrow radio pulses with duration smaller than 2nS. This leads to radical improvement of the most important radar parameters as range resolution, abilities for recognizing of targets and discovering small objects on background of large obstacles. These positive results are obtained by worth of considerable increasing of receiver complexity (for instance the used frequency band is wider than 500 MHz) [2]. Fortunately, this shortcoming is not critical due to the enhanced technical and economical parameters of present microelectronics element base.

With regard to positive features of nonsinusoidal radars, they are in very rapid progress in both theoretical and practical aspects recently. Despite of taken efforts, some important problems in the design of ultra wide band (UWB) nonsinusoidal radars are still open [1], [2]. Due to this reason our paper aims to suggest an effective computer algorithm for synthesis of nonsinusoidal radar signals (NRSs).

Our paper is organized as follows. First, a criterion for optimality of NRSs is considered. After then, an algebraic method for synthesis of NRSs is suggested. Finally, the advantages and possible areas of application of our method are discussed.

AN ALGORITHM FOR SYNTHESIS OF NONSINUSOIDAL RADAR SIGNALS

As mentioned above [1], [2], the transmitter of the nonsinusoidal radar radiates a nonsinusoidal signal, which is a sequence of ***w*** very narrow equivalent radio pulses without a carrier frequency. The sent signals are scattered by targets (objects) and small part of their energy returns back to the radar, where these so-named "echo-signals" are amplified in the receiver. In common case the echo signals are a diminishing copy of the sent signals. This fact is clarified on Fig. 1a, where an echo NRS, consisting (for convenience) of only 4 pulses, is shown. Time interval between any two elementary pulses of a NRS is proportional to the pulse duration τ_p. This limitation is introduced in order to simplify complex processes of generation and processing of NRSs by usage of digital microelectronic building elements. More specifically, in considered on Fig. 1 case $\tau_p = t_1 - t_0;\ \tau_1 = \tau_p;\ \tau_2 = 3.\tau_p;\ \tau_3 = 2.\tau_p$.

The receiver comprises an UWB amplifier which output is connected with ***w-1*** tapped delay-lines. They delay the amplified elementary pulses of an echo NRS at time intervals $\tau_{w-1}, \tau_{w-2}, ...\tau_2, \tau_1$, which are reversal to the order of sent pulses spacing. The outputs of the UWB amplifier and the delay-lines are simultaneously summed in an adder. As a result, the adder effectively calculates the auto-correlation function (ACF) of the received pulse sequences (see Fig. 1a, b, c, d, e). In presence of Gaussian noise this procedure maximizes the signal-to-noise ratio at the moment t_3 which is the arrival moment of the last pulse. Decisions for discovering of targets are taken by thresholding the adder output, named often "response of the receiver" (i. e. U_5 on Fig. 1). Suppose that radar is observing two targets. Then they will produce in the adder output two responses which are the ACFs of the NRS, reflected by the targets. If they are separated by interval, smaller than:

$$\Delta d = \frac{1}{2}\left(\tau_p + \sum_{i=1}^{w-1}\tau_i\right).c \qquad (1)$$

the receiver responses will overlap. Hence, the ACF side-lobes of more powerful echo signal, reflected by the larger target, will mask the main lobe of the ACF of the weaker signal, reflected by the smaller target. As a result, the smaller target, which often is more dangerous, will not be discovered. The probability of arising of this very undesirable situation, named "erroneous missing of the target", can be minimized only if the ACF side-lobes of the used NRS are as small as possible. It is straightforward that all side lobes will be zero only if $w = 0, 1$. Unfortunately similar NRSs are useless due to their small energy, which limits the radius of the space, observed by radar.

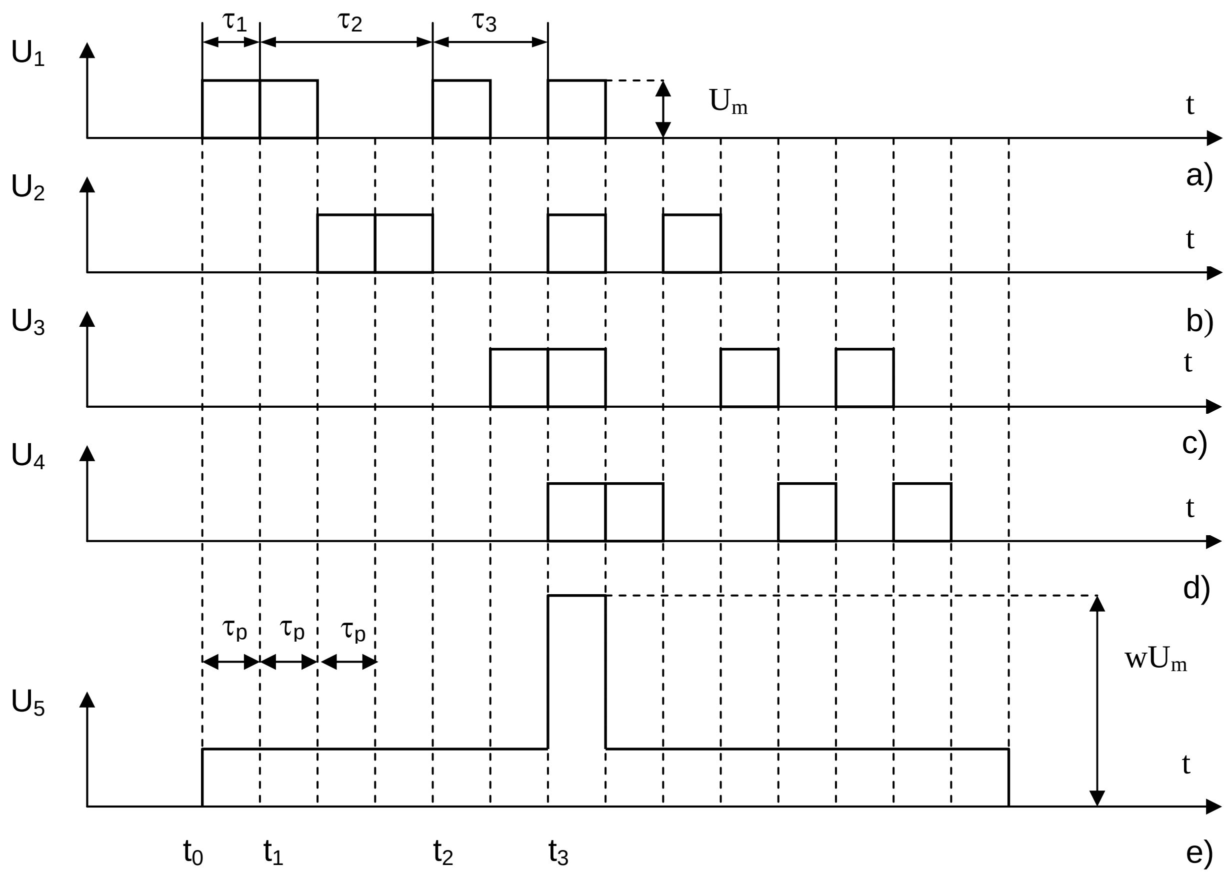

Fig.1: *Example of optimal nonsinusoidal radar signal (w=4)*

In (1):

$$T_{NRS} = \tau_p + \sum_{i=1}^{w-1}\tau_i \qquad (2)$$

is the duration of the NRSs, utilized by radar. Without losing of generality henceforth we shall assume that $U_m = 1[v]$ and we shall use the relative spacing (time-gap) between the elementary pulses:

$$\mu_i = \frac{\tau_i}{\tau_p},\ i = 1,2,...,w-1, \tag{3}$$

instead of absolute time-gaps τ_i, $i = 1,2,...,w-1$.

Above analysis shows that object of practical interest are only NRSs possessing both great number of pulses ***w*** and ACF with small level of side-lobes. Despite of taken efforts the problem of synthesis of such NRSs is still open [1], [2]. With regard, in the rest part of our paper we shall suggest a computational algorithm for synthesis of NRSs, which are optimal in the sense of the following criterion.

Criterion: A NRS, consisting of ***w*** elementary pulses with amplitude U_m, will be named optimal if:

- the maximum side-lobes of its ACF is U_m;
- the ACF main peak – to – signal length ratio:

$$\varepsilon = \frac{w}{N},\quad N = 1 + \mu_1 + \mu_2 + ... + \mu_{w-1} \tag{4}$$

is maximal for a fixed value of ***N***.

Now we shall prove the following Proposition 1, giving an important necessary condition a NRS to be optimal.

Proposition 1: A NRS is optimal if and only if all successive sums:

$$S(k,m) = \sum_{i=k}^{m} \mu_i,\ k = 1,2,...w-1,\ \ m = k,k+1,...,w-1 \tag{5}$$

are different.

Proof: The proposition will be proved by means of so-named "method of formal polynomials (or formal power sums of a single variable)" [3]. According to this method, a NRS, consisting of ***w*** elementary unity pulses, put in the relative time-positions:

$$\eta_0 = 0;\ \eta_1 = \mu_1;\ \eta_2 = \mu_1 + \mu_2;\ \eta_3 = \mu_1 + \mu_2 + \mu_3,\ ...\eta_{w-1} = \mu_1 + \mu_2 + ... + \mu_{w-1}, \tag{6}$$

is presented by the polynomial:

$$F(x) = x^{\eta_{w-1}} + x^{\eta_{w-2}} + ... + x^{\eta_2} + x^{\eta_1} + 1. \tag{7}$$

Then the lobes of the ACF of the NRS are the coefficients of the polynomial:

$$P(x) = F(x).F(x^{-1}), \tag{8}$$

where:

$$F(x) = x^{-\eta_{w-1}} + x^{-\eta_{w-2}} + ... + x^{-\eta_2} + x^{-\eta_1} + 1. \tag{9}$$

After evaluation of Eq. (8) the result is:

$$P(x) = \sum_{k=1}^{w-1}\sum_{m=k}^{w-1} x^{-S(k,m)} + w + \sum_{k=1}^{w-1}\sum_{m=k}^{w-1} x^{S(k,m)}\ . \tag{10}$$

The coefficients of the Eq. (10) right side, presenting the ACF side-lobes of the considered NRS, will be smaller or equal to 1 only if all sums $S(k,m)$ are different. This conclusion ends the proof of Proposition 1.

From Proposition 1 follows that all relative time-gaps of an optimal NRS must be different. Then the length of the NRS will be minimal if the time-gaps $\{\mu_1, \mu_2, ...\mu_{w-1}\}$ between elementary unity pulses are a permutation of the set $\{1, 2, ...w-1\}$. Having in mind that:

$$1 + 2 + ... + w = \frac{(w-1)w}{2} \tag{11}$$

the maximal value of the ACF main peak – to – signal length ratio is:

$$\varepsilon = \frac{w}{N} = \frac{w}{\left\lceil \frac{(w-1)w}{2} + 1 \right\rceil}. \qquad (12)$$

According to all above stated it is straightforward the following Proposition 2.

Proposition 2: A NRS, consisting of ***w*** elementary pulses, is optimal if and only if:

- time-gaps between the elementary pulses form a sequence $\{\mu_1, \mu_2, ..., \mu_{w-1}\}$ such that all successive sums $S(k,m)$ are different;

- the positive integers $\{\mu_1, \mu_2, ..., \mu_{w-1}\}$ are as small as possible.

In the rest part of our paper we shall suggest an computational algorithm, which satisfies the conditions of Proposition 2. It will be explained in more details by Fig. 2.

The main idea of the algorithm for optimal NRS synthesis is the sequence $\{\mu_1, \mu_2, ..., \mu_{w-1}\}$ ***to be successively filled up by putting "step-by-step" of appropriate new values in its middle. With regard, the algorithm consists of the following steps:***

1) In the initial step it is assumed $w = 2;\ \mu_1 = 1$. One can easily check that this NRS with three elementary pulses, placed in the time-positions $t_0 = 0.\tau_p; t_1 = 1.\tau_p$, is optimal. The beginning sum $S(k,m) = \sum_{i=k}^{m} \mu_i,\ k = m = 1$ is stored in a massive ***V₁(1,1)***, containing "the forbidden" values (which must not be used at the next steps). This massive is filling up during the every step by the rule:

$$v(k,m) = S(k,m),\ \ 1 \le k \le m \le n. \qquad (13)$$

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1	4	6
	3	5
		2

a) Matrix ***V***
n=3

1	4	9	11
	3	8	10
		5	7
			2

b) Matrix ***V***
n=4

1	4	10	21	40	64						
	3	9	20	39	63						
		6	17	36	60						
			11	30	54						
				19	43	68					
					24	49	72				
						25	48	62	70	75	77
							23	37	45	50	52
								14	22	27	29
									8	13	15
										5	7
											2

c) Matrix ***V***
n=12

Fig. 2: *Successive procedures, performed by the algorithm for optimal NRS synthesis*

n) During the ***n-th*** step, the following operations are performed:

n.1) The position $\left\lceil \frac{n}{2} \right\rceil + 1$ in the middle of the sequence $\{\mu_1, \mu_2, ..., \mu_{n-1}\}$ is set free. Here the symbol "$\left\lceil \frac{n}{2} \right\rceil$" denotes "the smallest positive integer equal or greater than $\frac{n}{2}$". More specifically:

$$\mu_n = \mu_{n-1},\ \mu_{n-1} = \mu_{n-2}, \ldots \mu_{\left\lceil \frac{n}{2} \right\rceil + 2} = \mu_{\left\lceil \frac{n}{2} \right\rceil + 1},\ \mu_{\left\lceil \frac{n}{2} \right\rceil + 1} = 0\,. \quad (14)$$

n.2) It is taken:

$$\mu_{\left\lceil \frac{n}{2} \right\rceil + 1} = \max\left\{ \mu_{\left\lceil \frac{n}{2} \right\rceil}, \mu_{\left\lceil \frac{n}{2} \right\rceil + 2},\ z \right\}, \quad (15)$$

where ***z*** is the minimal positive integer, which does not belong to the massive of forbidden values ***V_{n-1}(n-1,n-1)***, formed at the previous ***n-1-th*** step.

n.3) It is examined if all successive sums, which include $\mu_{\left\lceil \frac{n}{2} \right\rceil + 1}$, i.e. $S(k,m) = \sum_{i=k}^{m} \mu_i,\ 1 \le k \le \left\lceil \frac{n}{2} \right\rceil + 1 \le m \le w-1$, belong to the massive ***V_{n-1}(n-1,n-1)***. If the result of the check is:

n.3.1) ***"No"***, then:

- the current value $\mu_{\left\lceil \frac{n}{2} \right\rceil + 1}$ is accepted;

- the massive of forbidden values ***V_{n-1}(n-1,n-1)*** is filled up with all successive sums, including $\mu_{\left\lceil \frac{n}{2} \right\rceil + 1}$, which transforms it in ***V_n(n,n)***;

- the algorithm goes to the next ***n+1-th*** step, which is analogous to ***n-th*** step.

n.3.2) ***"Yes"***, then $\mu_{\left\lceil \frac{n}{2} \right\rceil + 1} = \mu_{\left\lceil \frac{n}{2} \right\rceil + 1} + 1$ and step n.3 is repeated.

The procedures at ***n-th*** step are clarified on Fig. 3. More specifically, Fig. 3a, b explain the case $n = 4$. As shown, first the position $\left\lceil \frac{4}{2} \right\rceil + 1 = 3$ in the middle of the sequence $\{\mu_1 = 1,\ \mu_2 = 3,\ \mu_3 = 2\}$ is set free. According to (14) this leads to the sequence $\{\mu_1 = 1,\ \mu_2 = 3,\ \mu_3 = ?,\ \mu_4 = 2\}$, where the symbol "?" denotes that μ_3 is unknown (free) still. Second, $\mu_3 = 5$ is taken, because ***5*** is the smallest integer, which does not belong to the massive of forbidden values ***V_3(3,3)*** (i.e. $z = 5$) and $\mu_2 < 5,\ \mu_4 < 5$. After that it is examined if all successive sums, which include μ_3, belong to ***V_3(3,3)***. In Fig. 3b this massive and all successive sums, which include μ_3, are depicted as upper triangle part of the matrix ***V***. All successive sums, which include μ_3, are located in the upper right sub-matrix, limited by darkened lines on Fig. 3b and the massive of forbidden values ***V_3(3,3)*** comprises entries $v(1,1) = 1,\ v(1,2) = 4,\ v(2,2) = 3,\ v(4,4) = 2$. Since considered upper entries of the matrix ***V*** are different, the value $\mu_3 = 5$ is accepted. It is not hard to examine that the sequence with elementary pulses, placed in time-positions $t_0 = 0.\tau_p;\ t_1 = 1.\tau_p,\ t_2 = 4.\tau_p,\ t_3 = 9.\tau_p,\ t_4 = 11.\tau_p$, is an optimal NRS. The 4^{th} step ends by adding the third row $v(3,3) = 5,\ v(3,4) = 7$ to the massive of forbidden values ***V_3(3,3)***, which transforms ***V_3(3,3)*** in ***V_4(4,4)***.

The computational effectiveness of above described algorithm for NRS synthesis is result of the following reasons:

1) The massive of forbidden values is evaluated successively.

2) The amount of numbers which must be checked in order to verify their acceptability as $\mu_{\left\lceil \frac{n}{2} \right\rceil + 1}$ is small. This is explained on Fig. 3c, where the case $n = 12$ is depicted. As shown, only the successive sums $S(6,7) = v(6,7) = 49,\ S(5,7) = v(5,7) = 68$,

$S(7,8)=v(7,8)=48$, $S(7,9)=v(7,9)=62$, $S(7,10)=v(7,10)=70$ must be examined. This is due to the inequalities $S(5,7)>S(8,12)=52$, $S(7,10)>S(1,6)=64$. First of them guarantees that the successive sums, ending with $\mu_{\lceil \frac{n}{2} \rceil +1}$ (excluding $S(6,7)$) are bigger that all successive sums, beginning with $\mu_{\lceil \frac{n}{2} \rceil +2}$. Analogously, the later inequality guarantees that the successive sums, beginning with $\mu_{\lceil \frac{n}{2} \rceil +1}$ (excluding $S(7,8), S(7,9)$) are bigger that all successive sums, ending with $\mu_{\lceil \frac{n}{2} \rceil}$.

Consequently, the entries $S(7,11), S(7,12)$ are evaluated due to necessity to add the 7th row $S(7,8), S(7,9), S(7,10), S(7,11), S(7,12)$ to massive $V_{11}(11,11)$ in order to be transformed in $V_{12}(12,12)$.

CONCLUSIONS AND FUTURE WORK

The above described algorithm for NRS synthesis was implemented in a computer program, working in C++ environment. With it all optimal NRS, possessing up to 1000 elementary unity pulses, were evaluated. The obtained results confirmed the computational effectiveness of our algorithm.

The suggested in the paper algorithm could be very useful in the process of optical orthogonal codes, which have close relationship with NRSs.

Nevertheless we intend to prove by more rigorous mathematical tools in a future work that our algorithm satisfies the second condition of Proposition 2.

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