A Method for Computer Synthesis of Radio Signals for the Next Generation Personal Communication Systems

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Abstract: *The spread spectrum signals have large application in modern personal communication systems, but they must meet many requirements such as: complex pseudorandom structure, ideal autocorrelation function, close to zero cross-correlation function for all of the signals used in the communication system. With regard to these requirements, recently some theorists proposed a new architecture for the next generation wideband wireless communications, based on so named Orthogonal Complementary Codes (OCC). The most distinguishable advantage of this architecture is possibility to operate without multiple access interference. As a result, the system capacity, secrecy, and electromagnetic compatibility can be improved significantly. Due to positive features of the OCC in a previous paper we generalized the OCC conception. On this base, in the present paper we prove a method for creating of generalized OCC with unlimited code length. It is realized as a computer program which automates all procedures during the OCC synthesis.*

Key words: *Computing, Computer Modelling, Complementary Sequences, Communication Systems.*

INTRODUCTION

The fast growth and the increasing number and quality of services offered by wireless communication systems require extremely optimal using the electromagnetic spectrum. A promising approach to solving of this hard technical problem is applying of so-named *Code Division Multiple Access (CDMA)* to the system resources. The most important positive features of *CDMA* systems are: resistance to negative effects of multipath wave spreading, invulnerability to unauthorized access, low interference with other electromagnetic sources and etc. These valuable abilities are result of that the radio signals are transmitted in a frequency band much wider than the minimal band required for sending the information. The spreading of the frequency spectrum over a wider band is reached by phase and/or frequency manipulating of the radio signals.

The spread spectrum signals must meet many requirements. The most important of them are: complex pseudorandom structure, ideal *autocorrelation function (ACF)*, similar to delta impulse, close to zero *cross-correlation function (CCF)* for all of the signals used in the communication system. With regard to these requirements, recently Chinese, Japanese and Russian authors have proposed a new *CDMA* architecture for the next generation wideband wireless communications, based on so named *Orthogonal Complementary Codes (OCC)* [2], [3]. The most distinguishable advantage of this architecture is possibility to operate without multiple access interference. As a result, the system capacity, secrecy, and electromagnetic compatibility can be improved significantly.

The classical *OCC* uses only binary phase manipulation. This constrains the control of the transmitted information rate. With regard to positive and negative features of the classical OCC, in an previous paper [1] we introduced the so - named *generalized orthogonal complementary codes (GOCC)*, which phase can obtain *m* ($m \geq 2$) values. On this base in the present paper we prove a method for creating of generalized *OCC (GOCC)* with unlimited code length. It is realized as a computer program which automates all procedures during the *GOCC* synthesis.

The paper is organized as follows. First, the basics of the *GOCC* are recalled. Second, a common method for synthesis of *GOCC* for personal communication system with arbitrary number of users is presented. After then a program for automated *GOCC* synthesis is explained. Finally, the advantages and possible areas of application of our method are discussed.

A METHOD FOR SYNTHESIS OF GENERALIZED COMPLEMENTARY CODES

The most distinguish inconvenience of the classical *OCC* is usage of only binary phase manipulation. This constrains the control of the transmitted information rate, which is an important procedure in present communication systems, including the wireless computer networks. With regard to this, first we will introduce the so - named *generalized orthogonal complementary codes (GOCC)* [1], which phase can obtain *m* ($m \geq 2$) values.

Referring to [1], the *GOCC* are set of *n* matrices $A_k,\ k = 1, 2, ..., n$, which entries $a_{lj}^k,\ \left|a_{lj}^k\right| = 1,\ k = 1, 2, ..., n;\ l = 1, 2, ..., n;\ j = 1, 2, ..., n$ are only *m*th roots of unity:

$$a_{lj}^k \in \{\exp(2\pi si / m);\ s = 0, 1, ..., m-1\} \tag{1}$$

and *the cross-correlation function (CCF)* of two arbitrary matrices in the set is:

$$R_{ks}(r) = \sum_{l=1}^{n} \sum_{j=1}^{n-r} a_{lj}^k . \tilde{a}_{lj+r}^s = \begin{cases} n^2 \delta(r),\ if\ k = s; \\ 0,\ if\ k \neq s, \end{cases} . \tag{2}$$

In Eq. (2):

- $r = 0, 1, 2, ...n-1$ is the horizontal shift of the matrix A_s relatively to matrix A_k;
- $\delta(r)$ is the Kroneker symbol:

$$\delta(r) = \begin{cases} 1,\ if\ r = 0; \\ 0,\ if\ r \neq 0; \end{cases} \tag{3}$$

- the symbol "~" means "*complex conjugation*".

Every matrix A_k describes mathematically the complex frequency and phase manipulated signal assigned to the *k*$^{-th}$ *CDMA* system user. Namely:

- the system frequency band *F* is divided into *n* subbands;
- every complex signal consists of *n* distinct frequency signals with carriers $f_l,\ l = 1, 2, ...n$, which are the central frequencies of the subbands;
- the phase of every carrier frequency is manipulated according to the elements of the rows of the *k*$^{-th}$ matrix A_k;
- the duration of an elementary phase impulse is τ.

It is necessary to underline two facts:

- amplitude modulation isn't used and hence the amplitudes of the carrier frequencies f_l are equivalent to U_m;
- the Eq. (2) shows that *the auto-correlation function (ACF)* of every signal has ideal shape, similar to a delta-pulse, and the *CCF* of every possible pair of signals is zero.

The positive features of the *GOCC* will be clarified with the following example.

Example 1: Let we suppose that the CDMA system has $n = 3$ users. Then the system frequency band is divided into 3 subbands which central frequency are f_1, f_2, f_3. To the three users are assigned complex radio signals, which are *GOCC* described with following three matrices:

$$A_1 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix};\ A_2 = \begin{bmatrix} 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \\ 1 & 1 & 1 \end{bmatrix};\ A_3 = \begin{bmatrix} 1 & \omega^2 & \omega \\ 1 & 1 & 1 \\ 1 & \omega & \omega^2 \end{bmatrix}. \tag{4}$$

where $m = 3$ and hence $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$; $\omega^2 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$, $1 + \omega + \omega^2 = 0$, $\tilde{\omega} = \omega^2; (\tilde{\omega}^2) = \omega$.

The *l*th ($l = 1, 2, 3$) row AR_l^k of the matrices A_k ($k = 1, 2, 3$) depicts the law of phase manipulation of the corresponding frequency carrier $f_l,\ l = 1, 2, 3$.

A *CDMA* system with *GOCC* works as follow. In the transmitter of the *u*th user every bit of the information input has duration T. It is multiplied in *n* subchannels with the corresponding rows of the matrix A_k. Consequently, if the input data has information velocity rate *C*, the derivative sequence of chips (which duration is *T/n*) has information velocity rate *nC*. After multiplication, the frequency carriers $f_l,\ l = 1, 2, ..., n$ are *m*-ary phase manipulated, according to the sequences, obtained in the l^{th} subchannel. The other users of the system receive the signals, sent from u^{th} user. In the receiver of the s^{th} user, a set of subband frequency filters separates the complex frequency signals in *n* subchannels. After that, in every subchannel signals are phase demodulated and aggregated. If the message is directed to the s^{th} user, i. e. $k = s$, then the result of summation is a sequence of *m*-its, which is very close to the transmitted sequence. Otherwise, if the data isn't for the s^{th} user, i. e. $k \neq s$, the sum is zero at all time of data transmission. These situations are shown on Fig. 1 (where *n*=*m*=*3*). It is easy to see, that the side-lobes of the auto-correlation and cross-correlation functions of rows of the matrices A_k, $k = 1,2,3$ mostly aren't zero, but after summation, the aggregated side-lobes vanish for every non zero time shift. In the case when the time shift is zero, then the aggregated central peak is:

- zero if modulating and demodulating matrices are different;
- $n^2.U_m$ if modulating and demodulating matrices coincide.

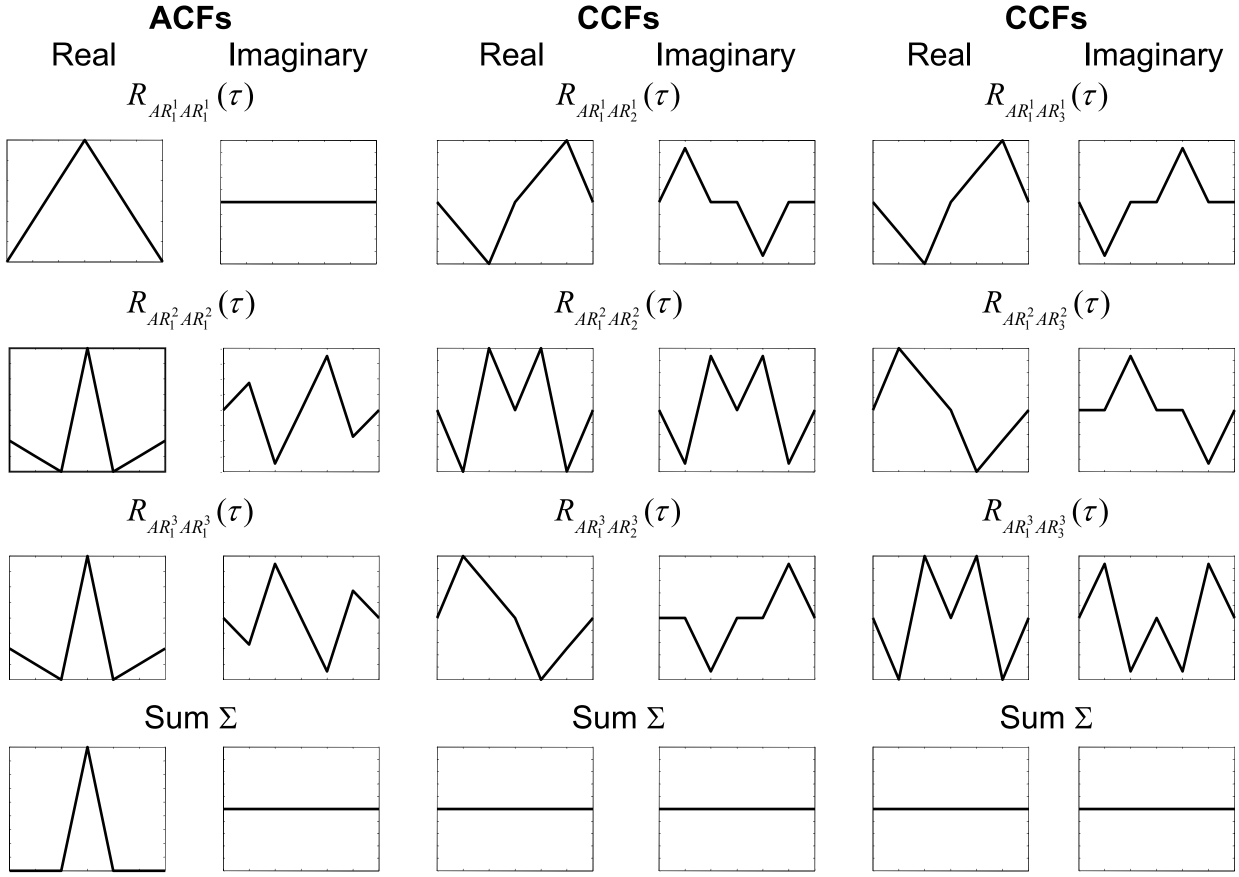

***Figure 1**:* Auto- and cross-correlation functions of the matrices $A_k\ (k = 1,2,3)$

Now we shall prove a sequence of corollaries and theorems which leads to our common method for synthesis of GOCC. At the beginning we shall recall the following Definition 1.

***Definition 1**:* The square matrix $G = (g_{jk})$ is called column orthogonal matrix if it satisfies the condition:

$$\tilde{G}^T.G = n.E, \qquad (5)$$

where n is number of rows and columns of the matrix G, E is the unity matrix (i.e. $e_{lj} = \delta(l-j),\ l = 1,2,...,n,\ \ j = 1,2,...,n$), the symbol "^{T}" means "transposition of the rows and columns" and the symbol "$\sim$" means "complex conjugation".

It is easy to see from Definition 1 that if G_p and G_q are two arbitrary columns of G, then:

$$G_p \otimes \widetilde{G}_q = \sum_{j=1}^{n} g_{jp}.\widetilde{g}_{jq} = \begin{cases} 0, & p \neq q; \\ n, & p = q. \end{cases}$$

(6)

In (6) the symbol "$\otimes$" means "*scalar multiplication of the vector – columns* G_p *and* $\widetilde{G}_q$".

Corollary 1: If the square matrix $G = (g_{jk})$ is column orthogonal matrix then it is row orthogonal matrix also.

Proof: From (5) can be concluded that:

$$G\widetilde{G}^T.G = Gn.E = nG.E = nG = (nE).G. \qquad (7)$$

Hence $G\widetilde{G}^T = nE$ what must be demonstrated.

Due to Corollary 1, in the rest part of the paper *the column orthogonal matrices* will be called simply *orthogonal matrices.*

Theorem 1: If the square matrix $A = (a_{jk})$ is an orthogonal matrix then the aggregated *ACF* of its rows satisfies the equation:

$$R_A(r) = \sum_{l=1}^{n} R_l(r) = n^2\delta(r), \qquad (8)$$

where:

$$R_l(r) = \sum_{j=1}^{n-r} a_{lj}.\widetilde{a}_{lj+r}, \qquad (9)$$

is the *ACF* of the l^{th} row.

Proof: From (9) Eq.(8) can be verified as follows:

$$R(r) = \sum_{l=1}^{n}\sum_{j=1}^{n-r} a_{lj}.\widetilde{a}_{lj+r} = \sum_{j=1}^{n-r}\sum_{l=1}^{n} a_{lj}.\widetilde{a}_{lj+r} = \sum_{j=1}^{n-r} B_j \otimes \widetilde{B}_{j+r} = n^2\delta(r). \qquad (10)$$

Here B_j, $\widetilde{B}_{j+r}$ are the j^{th} column and complex conjugated of the $(j+r)^{th}$ column of the matrix A respectively.

From an orthogonal matrix we shall create a set of matrices $C_1, C_2, ..., C_n$ by multiplying of the elements of a row with the column of the matrix A. Namely:

$$C_v = [a_{v1}.B_1,\ a_{v2}.B_2,\ ..., a_{vn}.B_n],\ \ v = 1,\ 2,\ ..., n. \qquad (11)$$

Theorem 2*:* The set of matrices $C_1, C_2, ..., C_n$ satisfies the Eq. (3), i. e. this set is *GOCC*.

Proof: Let we calculate the aggregated *CCF* $R_{ks}(r)$ of two arbitrary matrices C_k and C_s. Analogously to (10) we have:

$$R_{ks}(r) = \sum_{l=1}^{n}\sum_{j=1}^{n-r} c_{lj}^{k}.\widetilde{c}_{lj+r}^{s} = \sum_{l=1}^{n}\sum_{j=1}^{n-r}(a_{kj}.a_{lj}).(\widetilde{a}_{sj+r}.\widetilde{a}_{lj+r}) = \sum_{j=1}^{n-r}\sum_{l=1}^{n}(a_{kj}.a_{lj}).(\widetilde{a}_{sj+r}.\widetilde{a}_{lj+r}) =$$

$$= \sum_{j=1}^{n-r}\left[(a_{kj}.\widetilde{a}_{sj+r})\sum_{l=1}^{n}.a_{lj}.\widetilde{a}_{lj+r}\right] = \sum_{j=1}^{n-r}(a_{kj}.\widetilde{a}_{sj+r})\left(B_j \otimes \widetilde{B}_{j+r}\right) = \begin{cases} 0, & r \neq 0; \\ n.\sum_{j=1}^{n} a_{kj}.\widetilde{a}_{sj}, & r = 0. \end{cases} \qquad (12)$$

In fact (12) proves the Theorem 2, because according to Corollary 1 the matrix A is row orthogonal, i.e.:

$$n.\sum_{j=1}^{n} a_{kj}.\widetilde{a}_{sj} = \begin{cases} n^2, \; k = s; \\ 0, \; k \neq s. \end{cases}$$

The Theorem 2 shows the way how we can obtain *GOCC* if an orthogonal matrix $A = (a_{jk}); \forall a_{jk} \in \{\exp(2\pi si/m\}, \; m = 0, 1, ..., m-1$ is known. This is clarified with above Example 1, where $n = m = 3$, $A = A_1$ is chosen (see (4)) and the matrices A_2, A_3 are created according to (11).

In order to complete our common method for synthesis of GOCC we intend to demonstrate a construction for obtaining derivative orthogonal matrices with large sizes from orthogonal matrices with small sizes. Namely, we state the following Theorem 3.

Theorem 3*:* The product of Kronecker multiplication of two arbitrary orthogonal matrices *G* and *H* is an orthogonal matrix also.

Proof*:* Let orthogonal matrices G, H and product A of their Kroneker multiplication are:

$$G = [G_1, G_2, ..., G_p], \tag{13}$$

$$G_k = [g_{1k}, g_{2k}, ..., g_{pk}]^T, \tag{14}$$

$$H = \begin{bmatrix} h_{11} & h_{12} & . & h_{1q} \\ h_{21} & h_{22} & . & h_{2q} \\ . & . & . & . \\ h_{q1} & h_{q2} & . & h_{qq} \end{bmatrix}, \tag{15}$$

$$A = G * H = \begin{bmatrix} h_{11}G & h_{12}G & . & h_{1q}G \\ h_{21}G & h_{22}G & . & h_{2q}G \\ . & . & . & . \\ h_{q1}G & h_{q2}G & . & h_{qq}G \end{bmatrix}, \tag{16}$$

where the symbol "*" means "*Kroneker multiplication of the matrices G and H*".

According to (17) the scalar multiplication of two arbitrary columns of the matrix *A* is:

$$A_k \otimes \widetilde{A}_s = \sum_{j=1}^{q} \left(h_{jt}.G_u\right) \otimes \left(\widetilde{h}_{jv}.\widetilde{G}_w\right) = \sum_{j=1}^{q} \left(h_{jt}.\widetilde{h}_{jv}\right)\left(G_u \otimes \widetilde{G}_w\right) = p\sum_{j=1}^{q} \left(h_{jt}.\widetilde{h}_{jv}\right)\delta(u-w), \tag{17}$$

where:

$$k = (t-1).p + u, \;\; 1 \le t \le q, \;\; 1 \le t \le q, \tag{18}$$

$$s = (v-1).p + w, \;\; 1 \le v \le q, \;\; 1 \le w \le q. \tag{19}$$

If $u = w$, then follows:

$$A_k \otimes \widetilde{A}_s = pH_t \otimes \widetilde{H}_v = pq.\delta(t-v), \tag{20}$$

which competes the proof.

Theorem 3 allows creating of *GOCC* with unlimited code length. For instance, if $H = G = A_1$, $H = G = A_2$, or $H = G = A_3$ where A_1, A_2, A_3 are given in (4), after Theorem 3 *s* times applying, *GOCC* for a *CDMA* system with $n = 3^s$ users will be obtained. Moreover, it is possible to create initial matrices with appropriate value of *m*. Namely we intend to prove the following corollary.

Corollary 2**:** *Let:*

$$e = \exp(2\pi vi/m); \;\; g_{jk} = e^{(j-1)(k-1)}; \;\; j = 0, 1, ..., m-1, \; k = 0, 1, ..., m-1, \tag{21}$$

where $\gcd(v, m) = 1$. *Then the square matrix* $G = (g_{jk})$ *is an orthogonal matrix.*

Proof*:* The calculation of the Kroneker multiplication of two arbitrary columns of the matrix *G* shows:

$$G_k \otimes \widetilde{G}_s = \sum_{j=1}^{m} e^{v(j-1)(k-1)}.e^{-v(j-1)(s-1)} = e^{(k-s)}\left[\sum_{j=1}^{m} e^{v(j-1)}\right] = \begin{cases} \dfrac{e^{mv}-1}{e-1} = 0, \; if \;\; k \neq s; \\ \quad m; \; if \;\; k = s, \end{cases}. \qquad (22)$$

In fact the end result in (22) proves the corollary.

The Corollary 2 give a simple and useful method for finding orthogonal matrices, which elements satisfy (1) and phase modulation can be arbitrary (i.e. $m \geq 2$).

On the base of Theorem 3 and Corollary 2 we have realized a computer program which automates all procedures during the *GOCC* synthesis.

CONCLUSIONS AND FUTURE WORK

From all the above stated, it is easy to see that our method of synthesis of *GOCC*:

- generalizes the synthesis of the classical *OCC*, because the *OCC* are particular case of the introduced in the paper *GOCC*;

- can be applied successfully in the new generation *CDMA* communication systems, due to possibility to control the transmitted information rate by varying the value of *m*, which can be arbitrary according to Corollary 2.

It is necessary to emphasize that *GOCC* can be used in optical recognition systems also. In this case the numbers of matrix, denoting the *GOCC*, describe how is "painted" the colour marks over the targets [4], [5].

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