Transition matrix generation

Anatoliy Antonov, Yanka Yanakieva

Abstract: *The paper considers a new approach for regression based construction of transition matrix using market spread curves or historic probabilities of default. The generated transition matrix could be adjusted according to the credit year quality and the systematic component or using an aggregation schema.*
Key words: *Credit Risk, Probability of default, Transition matrix*

INTRODUCTION

The calculation of the Credit Risk of a Counterpart according to The Basle II Capital Accord involves estimation of Probability of Default (PD) that could be derived from corresponding transition matrix. The transition matrix represents moving probabilities from one rating level to all other rating levels within selected rating agency&system and time stamp for the period of one year. The last column of the transition matrix represents probabilities of default. Rating agencies (Standard&Pure, Moody,….) provide world wide valid transition matrices, which could not be directly applied for specific markets or industry branches. If a national bank institution intends to rate the credit standing of counterpart within own rating system a procedure for synthetic transition matrix generation is needed. The proposed procedure is based on Cumulative Default Probability Matrix, composed of the D-columns (default probabilities) of the Transition Matrixes for 2, 3, ... years, which are obtained by powering the initial Transition Probability Matrix for 1 year. The initial Transition Probability Matrix for 1 year is mapped with the historic Default probability matrix where a best-fit could be obtained using regression.

Emergency markets do not have statistic data for default probabilities, they are unstable and risky, so the best way to construct the needed Cumulative Default Probability Matrix is to use market spreads of government bond prices.

1. CUMULATIVE DEFAULT PROBABILITY MATRIX

Lower prices of Bonds on emergency market imply higher yields to maturity. A relevant part of that high yields represents expectation of market participants for credit risk, the other part reflects other types of risk such as liquidity, operational, FX, etc. Implied spreads for different maturity bands within a set of rating classes can be obtained using the high yields and the risk free yields. These spreads cause loses of bond value because of stronger discounting and imply in that way corresponding cumulative and marginal default probabilities and transition matrix that can be used to calculate expected credit losses within margin and contribution calculation for loans in the emergency market. The methodology of calculating an implied transition matrix from bond prices on emergency market include following steps:

- Start point is an emergency market bond Portfolio with actual market prices. The maturity of bonds should cover standardized maturity bands around maturity points such as 1, 2, 3, 5, 7,... years. The bonds should be rated within a set of rating levels such as 1,2,3,...,9.
- The Portfolio is then structured into subportfolios according to rating classes and maturity bands within every rating class. The next step is calculation of all subportfolios and obtaining aggregated yields to maturity for every maturity band within every rating class. Missing yields to maturity can be interpolated/extrapolated from existing yields. We obtain after this step a set of market yield curves, one curve for every rating class

- The risk free market curve can be calculated from an other benchmark Portfolio containing government bonds using similar approach as above or the risk free market curve can be obtained from provider.
- The next step is the calculation of the zero rates from the risk free market curve and from the market curves of the rating classes. This is a standard procedure that removes the coupon payments and calculates the zero coupon rates.
- The spread curves for the rating classes are obtained then using an adjustment for the spread implied by credit risk only:

  *Spread = Risk Free Zero + Adjustment * (Rating Zero – Risk Free Zero)*

  We obtain after this step a set of zero rate spread curves, one curve for every rating class.
- The zero rate spreads imply cumulative default probabilities. The same expected loss can be calculated using discounting of a payment with spreaded zero yield or using default probability (PD) and assumed Recovery Rate (RR):

  *Loss = 1 - 1/(1+Zero+Spread)^t = 1*PD(t)*(1-RR)/(1+Zero)^t*
  PD(t) = (1 – ((1+Zero)/(1+Zero+Spread))^t)/(1-RR) or
  *PD(t) = (1 – e^(-Spread*t))/(1-RR) for exponential discounting*

  We obtain after this step a set of default probability curves, one curve for every rating class. Marginal probabilities for every future period can be calculated from default probability curves using exponential interpolation and probability difference between period end and period begin.
- The last step is the estimation of the corresponding transition matrix. This step needs a regression procedure starting from a starter matrix with dimensions corresponding to rating classes. Cumulative default probabilities for longer periods can be obtained from last default column of powered starter transition matrix (T):

	1 year Transition Matrix							
From Rating	To Rating							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	88,658%	10,294%	1,017%	0,000%	0,031%	0,000%	0,000%	0,000%
AA	1,079%	88,705%	9,553%	0,342%	0,145%	0,145%	0,000%	0,031%
A	0,063%	2,876%	90,205%	5,919%	0,740%	0,177%	0,010%	0,010%
BBB	0,053%	0,339%	7,069%	85,238%	6,053%	1,005%	0,085%	0,159%
BB	0,033%	0,077%	0,557%	5,680%	83,572%	8,083%	0,535%	1,464%
B	0,011%	0,044%	0,174%	0,652%	6,595%	82,703%	2,760%	7,062%
CCC	0,000%	0,000%	0,660%	1,050%	3,050%	6,110%	62,970%	26,160%
D	0,000%	0,000%	0,000%	0,000%	0,000%	0,000%	0,000%	100,000%

T^1 is the starter matrix, T^2 is powered providing DP's for 2 years,
T^3 is powered providing DP's for 3 years and so on. Using the formulae

$$\left|\mathbf{T_i}\right| = \left|\mathbf{T_{i-1}}\right| \times \left|\mathbf{T_1}\right| \qquad \left|\mathbf{T_i}\right| = \left|\mathbf{T_1}\right|^{\mathbf{i}}$$

the Cumulative Default Probability Matrix is obtained:

Cumulative Default Probability Matrix							
	AAA	**AA**	**A**	**BBB**	**BB**	**B**	**CCC**
1 Year	0,000%	0,031%	0,010%	0,159%	1,464%	7,062%	26,160%
2 Years	0,004%	0,073%	0,056%	0,477%	3,407%	13,722%	43,111%
3 Years	0,012%	0,127%	0,145%	0,950%	5,678%	19,828%	54,255%
4 Years	0,027%	0,198%	0,284%	1,568%	8,157%	25,339%	61,720%
5 Years	0,050%	0,289%	0,477%	2,317%	10,750%	30,270%	66,840%

The regression procedure changes the starter matrix elements and compares the default probabilities from matrix powering and the default probabilities from implied default probability curves searching for "best fit". The last state of the starter transition matrix after finishing the regression is the desired 1 year transition matrix. The regression procedure uses an random number based search algorithm. An initial "ones" matrix multiplies the transition matrix, a set of random numbers changes the "ones" matrix. A set of restrictions (s. section 2.) is then applied to discard the unusual solutions.

2. REGRESSION RESTRICTIONS

The synthetic generation of implied 1 year transition matrix from cumulative default probabilities for 1, 2, 3, .. year(s) transition probability matrix using a starter matrix and a regression ("best fit") algorithm holds following regulatory restrictions:

- The sum of all transition probabilities for every rating level is calculated to be 100 %.
- The 1 year default probabilities from cumulative default probabilities are moved to 1 year default probabilities (column D) of starter matrix and retained while regression process.
- The default probabilities (column D) are checked to step monotony up with increasing the rating level.
- The transition probabilities for every rating level are checked to step monotony down to left and to right from corresponding rating level column that exposes a probability maximum. This check don't include the default probabilities (column D).
- The expected value of the probability distribution for every rating level is checked to fail into corresponding rating level column, that is the absolute value of:

  (expected value of the rating level number – rating level number) < 0,5

  This check don't include the default probabilities (column D). This restriction can be disabled if generating implied complex matrix from large rating spreads or if using very risky market data.

3. TRANSITION MATRIX ADJUSTMENT

The synthetic generation of transition matrix using "best-fit" procedure is based on starter matrix, which could be obtained (down loaded) from a rating agency for specific rating system. In case one need transition matrix for specific market within own rating system with lower rating level number the implied transition matrix can be adjusted using following approaches:

- Shifting the transition matrix down
- Aggregation of transition matrix rows/columns.

The Source Transition matrix is shifted down using a procedure described in CreditMetrics ® Monitor [1]. This step incorporates the Shifting of the whole transition matrix accounting for quality of credit year for given market or region.

The second approach requires that the user assigns an aggregation schemata which allows to aggregate rows/columns form shifted source matrix into desired result matrix. A matrix of larger dimensions is pasted into matrix of smaller dimensions on this way allowing for additional adjustment of default and transition probabilities. An example aggregation schemata adjusting the 18 level Standard & Pure transition matrix (SnP18) to a new 10 level one (AKAP10) is given below:

SnP18	AKAP10
AAA, AA+, AA	Set and aggregate to AKA1
AA-, A+, A	set and aggregate to AKA2
A-, BBB+	set and aggregate to AKA3
BBB, BBB-	set and aggregate to AKA4
BB+ BB	set and aggregate to AKA5
BB-, B+	set and aggregate to AKA6
B	set and aggregate to AKA7
B-	set and aggregate to AKA8
CCC	set and aggregate to AKA9

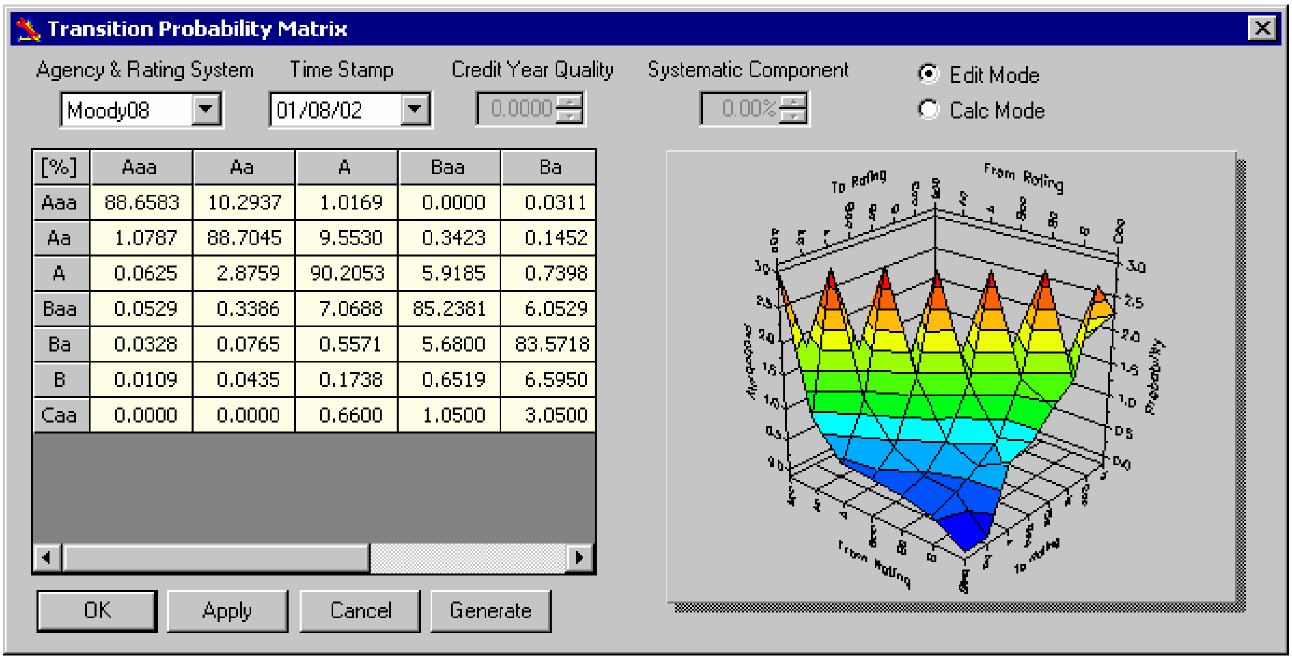
The aggregation of transition matrix rows/columns is performed by calculation of the mean of the aggregated rows/columns, for example Transition probability of AKA1 to AKA1 (AKA1 => AKA1) is obtained from the probabilities AAA, AA+, AA according to the aggregation schema by:

(Sum (AAA => AAA, AAA => AA+, AAA => AA) +
Sum (AA+ => AAA, AA+ => AA+, AA+ => AA) +
Sum (AA => AAA, AA => AA+, AA => AA)) / 3

4. IMPLEMENTATION

The transition matrix generation procedure using the described regression approach and the adjustment mechanism are implemented in the Software Package Risk Evaluator developed 2002 by Eurorisk Systems Ltd.. The Package includes the Downloader module intended for down loading of standard transition matrices published by rating agencies and the Data Supporter Module intended for preparation of credit risk data especially for transition matrix generation. The module could be used for ordinary and emerging markets providing following functionality :

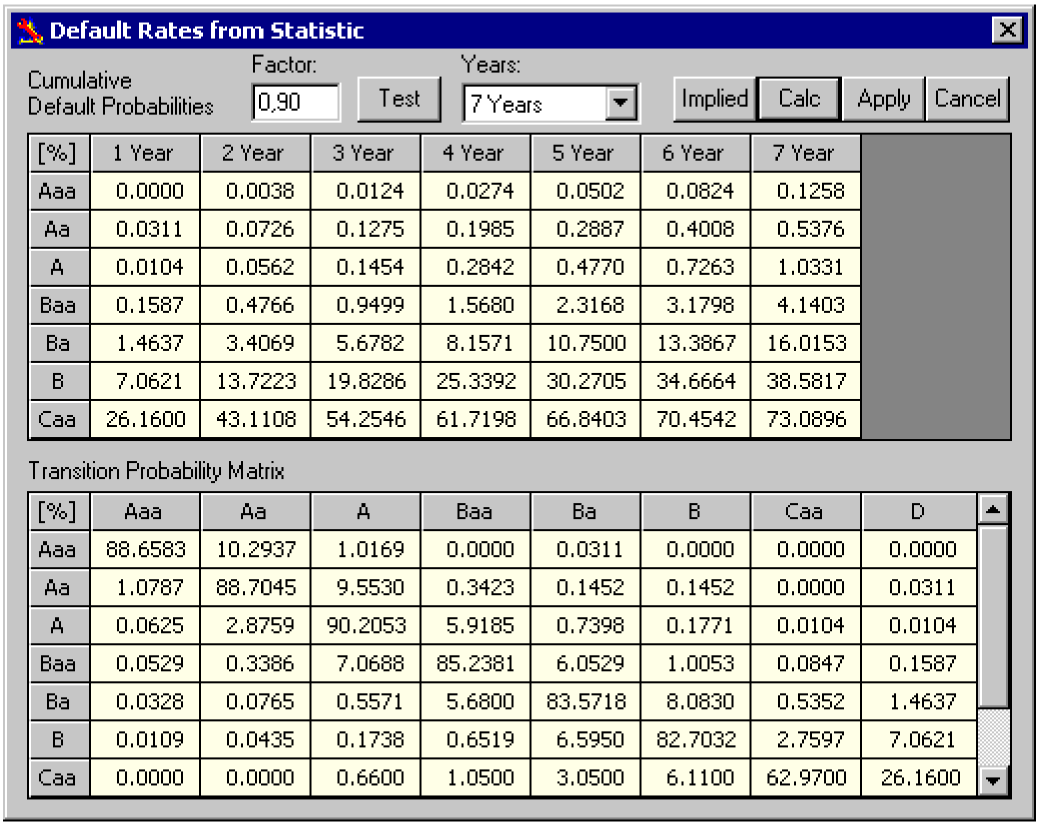
- manually editing a downloaded or generated Transition Matrix - Edit Mode (Fig. 1)
- calculating of new Transition Matrix using the selected values of Credit Year Quality and Systematic Component - Calc Mode (Fig. 1)
- generating an own Transition Matrix using historic default probabilities imported by Clipboard copy (Fig. 2)



[%]	Aaa	Aa	A	Baa	Ba
Aaa	88.6583	10.2937	1.0169	0.0000	0.0311
Aa	1.0787	88.7045	9.5530	0.3423	0.1452
A	0.0625	2.8759	90.2053	5.9185	0.7398
Baa	0.0529	0.3386	7.0688	85.2381	6.0529
Ba	0.0328	0.0765	0.5571	5.6800	83.5718
B	0.0109	0.0435	0.1738	0.6519	6.5950
Caa	0.0000	0.0000	0.6600	1.0500	3.0500

Fig.1: Editing and generating a transition matrix in the main Panel of Data Supporter

- generating an own Transition Matrix using spreads for the constructing of Cumulative Probability Matrix – Implied Button (Fig. 2)
- applying aggregation schema to rows/columns in order to obtain a smaller dimension matrix
- accepting the edited (calculated, synthetically generated) Transition Matrix
- saving the current Transition Matrix into the data base



[%]	1 Year	2 Year	3 Year	4 Year	5 Year	6 Year	7 Year
Aaa	0.0000	0.0038	0.0124	0.0274	0.0502	0.0824	0.1258
Aa	0.0311	0.0726	0.1275	0.1985	0.2887	0.4008	0.5376
A	0.0104	0.0562	0.1454	0.2842	0.4770	0.7263	1.0331
Baa	0.1587	0.4766	0.9499	1.5680	2.3168	3.1798	4.1403
Ba	1.4637	3.4069	5.6782	8.1571	10.7500	13.3867	16.0153
B	7.0621	13.7223	19.8286	25.3392	30.2705	34.6664	38.5817
Caa	26.1600	43.1108	54.2546	61.7198	66.8403	70.4542	73.0896

Transition Probability Matrix

[%]	Aaa	Aa	A	Baa	Ba	B	Caa	D
Aaa	88.6583	10.2937	1.0169	0.0000	0.0311	0.0000	0.0000	0.0000
Aa	1.0787	88.7045	9.5530	0.3423	0.1452	0.1452	0.0000	0.0311
A	0.0625	2.8759	90.2053	5.9185	0.7398	0.1771	0.0104	0.0104
Baa	0.0529	0.3386	7.0688	85.2381	6.0529	1.0053	0.0847	0.1587
Ba	0.0328	0.0765	0.5571	5.6800	83.5718	8.0830	0.5352	1.4637
B	0.0109	0.0435	0.1738	0.6519	6.5950	82.7032	2.7597	7.0621
Caa	0.0000	0.0000	0.6600	1.0500	3.0500	6.1100	62.9700	26.1600

Fig.2: Generating a new transition matrix using "best-fit" procedure

CONCLUSIONS AND FUTURE WORK

The transition matrix including probabilities to move from one rating to another rating represents the kernel of many credit risk and rating calculations. Following the requirements of Basle II financial engineers need software tools allowing for adjustment of transition matrices provided by rating agencies to the economic cycles and to generate transition matrices according to the local financial and economic conditions.

The generated transition matrixes are the basis for calculation of credit risk of counterparts using a set of internal (CreditMetrics, Credit Risk+) and Basel II (Standard, Foundation and Advanced Approaches) evaluation models. The estimation of credit risk involve cumulative and marginal default probabilities for future periods used to calculate expected and unexpected losses (Credit VaR) within a multi-period credit exposure model. The results of transition matrix estimation of Data Supporter Module are used by Risk Evaluator to calculate credit risk of single counterparts and various aggregates based on subportfolios, concern structures and other grouping criteria such as branches and countries.

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ABOUT THE AUTORS

Assoc.Prof. Anatoliy Stefanov Antonov, PhD, Department of Computer Systems and Technologies, Technical University of Varna, Phone: +359 52 612 367,
E-mail: antonov@euroriskystems.com
Yanka Nedelcheva Yanakieva, Department of Computer Systems and Technologies, Technical University of Varna,
E-mail: yanakieva@euroriskystems.com